Collateralization and asset price bubbles when investors disagree about risk

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Abstract

Survey respondents disagree strongly about the dispersion of future returns and, increasingly, macroeconomic uncertainty. Such disagreement about risk may raise asset prices when collateralized debt products allow investors to realize perceived gains from trade. Investors who expect low volatility in collateral cash flow appreciate senior debt as riskless. Those who expect high volatility, in contrast, value the upside potential in junior debt or equity claims. We show how such self-selection may have had a sizeable effect on the prices of RMBS and CDOs before the crisis, as investors disagreed about the volatility of aggregate economic conditions and their importance for default rates in collateral pools.

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Keywords: Asset Prices, Heterogeneous Beliefs, Disagreement, Volatility, Securitization, Structured Finance, Bubbles

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1 Introduction

From the mid-1990s to the beginning of the Great Recession, the world economy has seen an unprecedented wave of financial innovation, partly in the form of new collateralized debt products. At the same time, prices of collateral assets, such as real estate, but also stocks, experienced an unprecedented increase. This paper links these two phenomena to a third, less documented one: disagreement among investors about economic risk. We provide evidence for this argument from several US surveys. We first show how the data analyzed by `?AmrominSharpe12` () and `?BenDavidetal2013` () imply strong disagreement among both retail investors and finance professionals about the dispersion of future stock returns. Second, to analyze a longer time horizon covering the Great Moderation period, we document that, since the early 1990s, near-term GDP forecasts from the Survey of Professional Forecasters show increasing disagreement among forecasters about the dispersion of GDP growth, while disagreement about mean growth has fallen. We conclude from this that disagreement about risk is substantial, and that there is some evidence that it became more important relative to disagreement about mean payoffs in the 1990s and early 2000s.

We show how such heterogeneous risk perceptions, when combined with financial innovation in the form of collateralized debt products, can create asset price bubbles. In the absence of collateralization, risk-neutral investors trade assets at their common fundamental value even if they disagree about payoff risk. The introduction of risky collateralized debt products increases asset prices above this common fundamental value by splitting cash flow into senior debt and junior debt or equity claims. Investors who perceive low volatility are happy to pay high prices for senior debt, which they regard as riskless. Those who think volatility is high, in contrast, value the upside potential in junior claims. Disagreement about risk thus raises the equilibrium price of collateral assets as investors self-select into buying the claims they value most highly. We show how this may have been an important driver of the boom in 'Structured Finance' assets, such as collateral debt obligations (CDOs), whose senior tranches are attractive to investors who believe in diversification and thus think default rates of collateral pools are stable. Those, in contrast, who think default rates are more reflective of aggregate conditions, and thus more volatile, think senior tranches may still fail in bad times, but are happy to pay for junior and equity tranches, which they expect to pay when conditions are sufficiently good.

Our simple theoretical benchmark model focuses on a given mortgage pool whose cash flow can be traded by risk-neutral investors in the form of a debt and an equity tranche. The insight that collateralization increases asset prices, however, applies to any risky debt contract collateralized by a payoff whose dispersion investors disagree about.[1] In a quantitative application of our theory, we consider more complex debt instruments that split collateral cash-flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Such structured securities were blamed for their role in the US housing boom and the financial crisis that followed after their issuance had experienced a spectacular rise in the early 2000s. We find that even modest disagreement about the variability of default rates can raise the market value of a typical US residential mortgage backed security (RMBS) by more than 100 basis points above the expectation of collateral cash flow (that we assume is shared by all investors). This 'return to tranching' may be an order of magnitude larger still for RMBS-backed CDOs, whose payoff distributions are not bounded below by a minimum recovery value and thus more sensitive to changing perceptions of risk. Disagreement about risk may thus be an additional reason for the boom in Structured Finance during the early 2000s,[2] although the precise timing of that boom is likely explained by other factors pointed out in the literature.[3]

[1]An online appendix extends the theoretical result to environments with risk aversion (in Section ??). It also shows that, although collateralization splits cash flows in a way similar to trade in call or put options, disagreement about risk also raises asset prices with trade in (cash-collateralized) options (in Section ??).

[2]Other factors that contribute to the attractiveness of structured debt are the mitigation of information asymmetries and the creation of safe assets through issuance of (super-) senior tranches, regulatory arbitrage ?acharyaetal2013 () and ?brunnermeier2009 (), rating bias ?griffin2011 () and ?griffin2012 (), as well as investors' disregard of certain unlikely risks ?gennaiolietal2012 () or of their highly systemic nature ?Covaletal2009b ().

[3]Such factors include financial innovation and deregulation in the US that made it easier to collateralize large baskets of mortgage loans and other risky assets (?bozmendoza2014), the advent of a large pool of standardized high-risk collateral in the form of US subprime or Alt-A mortgages (attributed for example to the technological innovation in underwriting procedures, ?Gorton2009 (), ?Gatesetal2002 ()), the disintermediation of the US financial system (boosting the demand for repo collateral), changes in banking regulation (reducing the relative capital requirements for investments in senior securitization tranches), the low interest rate environment of the early 2000s, or the boost to the private-label RMBS market

Structured Finance is not the only asset class where our theory may be important. For example, our results have implications for the theory of firm financing: contrary to ?MM1958 (), they call for a mix of debt and equity finance that depends on the heterogeneity of risk perceptions in the investor pool. Specifically, firms optimally issue debt to investors who perceive risk to be low, and sell equity to those who perceive higher risk and thus stronger upside potential to shares in the firm.

Previous studies of disagreement have largely focused on disagreement about an asset's mean payoffs, where 'optimists' expect higher payoffs than 'pessimists' and, absent short-selling, drive prices above average valuations (?Miller1977). Leverage through riskless collateralized loans may raise prices further by increasing investment funds of optimists (?Geanakoplos2003). When risky collateralized debt can be issued (?Simsek2013), optimists face a trade-off: in order to raise funds for investment into upside risk, they have to sell downside risk at unfavorable prices to pessimists. With trade in collateralised debt contracts, the effect of disagreement about mean payoffs is thus dampened when optimists have positive views mainly about downside risk. Importantly, the asset price does not exceed optimist valuations unless more complex assets are traded or investors are borrowing constrained (?FostelGean08; ?FostelGeanakoplos12).

Disagreement about the dispersion of payoffs affects the price of collateral assets in a way that is fundamentally different. Investors who perceive asset payoffs to be more volatile than others are optimistic about upside risk at the same time as they are pessimistic about downside risk. Conversely, low-volatility investors are downside optimists and upside pessimists. By allowing them to trade up- and downside risk separately, risky collateralised debt leads to self-selection of investors into buying their preferred risks. This realizes pure gains from trade and raises prices above the *maximum* valuation of collateral assets. This is in contrast to disagreement about means, as e.g. in ?Simsek2013 (), where the optimistic valuation is typically an upper bound for the asset price. Relative to ?Simsek2013 (), we also provide explicit conditions in terms of

through the sale of mortgage portfolios by the US government after the savings and loans crisis of the late 1980s / early 1990s.The role of affordable housing policy in the subprime boom, in contrast, is controversial. On these points see e.g. the Financial Crisis Inquiry Commission's Report (?fcic2011), p. 68-80.

exogenous variables for an increase in disagreement to increase asset prices further. ?Simsek2013 ()'s Theorem 5, in contrast, states conditions that involve the endogenous face value $\bar{s}$. Similar to Example 2 in ?Simsek2013 (), our Example 1 illustrates that an increase in disagreement may indeed also lower asset prices. ?geerolf2018leverage () considers a continuum of investors in an environment similar to ?Simsek2013 ()'s but with point beliefs and shows how this implies in equilibrium a bilateral assignment of lenders and borrowers to collateralised loan contracts that differ in interest and face value.

Because we interpret the collateral asset as a pool of loans whose defaults investors perceive as more or less correlated, our theoretical analysis contributes to a recent literature that studies the effect of disagreement about default characteristics on prices of collateralized debt tranches. In particular, ?Broer2018 () discusses qualitatively the effects of disagreement about average default rates and their variability on the prices of structured finance assets using a simple two-loan example. The theoretical analysis of this paper studies disagreement about risk in a substantially richer environment with many collateral assets. Moreover, we also identify conditions for an increase in disagreement to increase the collateral price further (Proposition 3), and provide an example where it reduces the collateral price (Example 1). After our main analysis was complete, ?Ellisetal17 () show that structuring in tranches is the optimal security design within the set of monotone securities for general disagreement. They show that there exists a unique tranching equilibrium, and, like the theoretical part of this paper, provide conditions such that collateral prices exceed the valuations of all investors. Their environment, with a dedicated issuer choosing an optimal number of tranches, differs from that of our theoretical analysis with a simple two-tranche structure that makes the results immediately applicable to any setting where loans are collateralized by any risky asset. Moreover, and related to our empirical evidence, we discuss in detail when an increase in disagreement about risk raises collateral prices (and provide a simple, if extreme, example where it lowers them).[4] In related work, (?gong2020collateral) consider a general equilibrium model with heterogeneous investors and collateralised borrowing

[4] ?BianchiJehiel2016 () consider disagreement about average default probabilities in a similar context.

and show that when a risky asset can be tranched or when derivative contracts backed by the asset can be used as collateral for other assets ("pyramiding"), the equilibrium risk premium of the asset may be smaller than the price of insuring its risk (implying positive "basis").

Since the equity and debt tranches we consider are equivalent to, respectively, buying a call option and selling a put option on the payoff of the underlying loan portfolio, our analysis also relates to the literature on the effects of heterogeneous beliefs on options prices (?Li2007; ?li2013investors; ?osambela2015differences; ?Feng2015). Perhaps because volatility is perfectly observable for continuous-time Brownian motion, that literature abstracts from disagreement about volatility, while we focus on contexts where either sampling or the underlying shock process are discrete, which we think is particularly true for innovations originating from macroeconomic shocks. The next section presents evidence from US surveys that shows disagreement about return risk to be important, and disagreement about macroeconomic risks to have increased between 1990 and 2016, when it accounted for about two thirds of overall disagreement. Section III presents a simple general equilibrium model with two investor types, whose disagreement about risk leads to an asset price bubble when they can trade collateralized debt whose riskiness is determined endogenously. Section IV uses a quantitative model of structured loan pools to gauge the effect of disagreement about credit risk for the US mortgage market.

2 Evidence: Disagreement about risk in US surveys

One aim of this paper is to show how a reasonable amount of disagreement about risk can have an important impact on asset prices. This section provides evidence about the importance of such disagreement. A common problem in expectational surveys is respondent noise. We concentrate on respondents that are incentivized to have good information, namely investors with substantial stock investments (in the Michigan survey), financial executives (?BenDavidetal2013 ()), professional forecasters, and (in an online appendix) house owners. We therefore interpret heterogeneity in the dispersion of reported distributions as disagreement about risk, and not (rational) ignorance or agnosticism.

2.1 Disagreement about US stock market returns

This section uses information from two US surveys to show how investors strongly disagree not only about expected returns, but also about return risks. Table **??** reports summary statistics of the supplementary questions in the Michigan Survey of Consumer Sentiments, covering 22 surveys in the years 2000 to 2005, taken from ?AmrominSharpe12 ().[5]

The first row of Table **??** shows that expected annual returns, averaged across respondents and surveys, equal 10 percent, close to the average 10 year annual returns on the S&P total returns index in the period before the last survey in 2005. Disagreement about future mean returns, however, is strong, with 10 percent of respondents expecting an average return of or below 5, and another 10 percent expecting above 16 percent. The perceived riskiness of stock investments, however, also varies strongly across investors: while 10 percent of respondents believe realized returns to fall within 2 percentage points of their expectation with a probability of at least 80 percent, another 10 percent expect returns to fall outside this range with at least 80 percent probability.[6] Importantly for our analysis, these differences in expected dispersion of the stock market can be interpreted as disagreement about the correlation of individual stock prices, and is thus, again, indicative of heterogeneity in perceived correlation of asset prices.Using a normality assumption to transform these assessments into standard deviations, the 90-10 percentile difference of standard deviations equals 6.3, compared to 11 for expected returns.

?BenDavidetal2013 () present similar survey evidence for a sample of senior finance execu-

[5]The authors eliminate incomplete responses, those deemed by the interviewer to have a low "level of understanding" or a poor "attitude" towards the survey, and those that answered "50 percent" to all probability questions.

[6]The question asks for the probability "that the average return over the next 10 to 20 years will be within two percentage points of your guess". We interpret the heterogeneity in responses to this question as evidence of heterogeneous perceptions of risk. An alternative interpretation is that of heterogeneous confidence in individual point estimates.

Table 1: Return Expectations in the Michigan Survey 2000-2005

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	N	Mean	10th pct	25th pct	Median	75th pct	90th pct
Expected return R_e	3,046	10.4	5	7	10	12	16
Prob $\|R - R_e\| < 2pp$	3,015	43.3	20	25	50	50	80
Implied σ_{10-20} (in percent)	2,854	4.56	1.56	1.73	2.96	2.96	7.88

The first row reports the distribution of investors' answer to the question about the "annual rate of return that you would expect a broadly diversified portfolio of U.S. stocks to earn, on average". The second row reports the probability "that the average return over the next 10 to 20 years will be within two percentage points of your guess", and the third one shows the corresponding standard deviation assuming normally distributed beliefs about stock market returns.

tives, mainly Chief Financial Officers. They show how their respondents' forecasts of US S&P 500 returns are 'miscalibrated', in the sense that respondents underestimate the uncertainty around their expected returns both relative to history and relative to subsequent outcomes. Interestingly for the present study, they also show how respondents strongly disagree in their volatility estimates. For example, the individual standard deviations of 1-year-return forecasts implied by their survey responses have a distribution whose 95-5 percentile difference equals 15 percentage points (both for the whole sample from 2001 to 2011, and the 2011Q1 cross section). Interestingly, the disagreement about expected returns is similar to that about expected volatility both in terms of the standard deviation across respondents (which equals 5.3 for expected returns and 4.3 for standard deviations) and the 95-5 percentile difference (equal to 15 percentage points also for means).

2.2 Disagreement about US Macro Risk 1991-2016

The Survey of Professional Forecasters SPF is a quarterly survey that asks forecasters to indicate, among other measures, their probability distribution for GDP growth in the current calendar year. Importantly, its anonymous nature makes it a good source of information about actual forecaster expectations, as it presumably reduces concerns about strategic behavior (`?marinovic2013forecasters`). Forecasters report the probability that short-term growth falls in any of 6 brackets. This allows us to study the evolution of disagreement between forecasters about short-term US growth prospects. Specifically, using a normal approximation of the distributions, as in ?GiordaniSoderlind2003 (), we can look at the distribution across forecasters of forecaster-specific means μ_{it} and standard deviations σ_{it}, where i denotes forecasters and t the forecast period. Second, to assess the relative importance of means and standard deviations for overall forecaster disagreement, we look at a measure of total disagreement based on the integral of absolute differences of any two forecaster-specific normal densities f_i, f_j

$$d = \frac{1}{2}\frac{1}{N_t^2}\sum_i\sum_j \int |f_i(g_y) - f_j(g_y)| dg_y, \tag{1}$$

where N_t is the time-varying number of forecasters in the sample.[7] This measure allows us to calculate the contribution of the heterogeneity in standard deviations to this average disagreement using the formula in (**??**) with the mean of the two normal distributions held constant ($\mu_{it} = \mu_{jt}$). We then define the remaining difference with overall disagreement as the contribution of heterogeneous means.[8]

We limit our sample to the years 1991 to 2019.[11] Figure **??** depicts time series of our disagreement measures. To keep the forecasting horizon constant and equal to the remainder of the current year, we only use data collected during the first quarter of every year. The top left panel shows how the standard deviation of σ_i rose throughout the 1990s, and again after the beginning of the financial crisis in 2007. In contrast, the dispersion of mean forecasts (in the top right panel) showed no such rise, but rather declined persistently after its spike following the Great Recession. As an illustration of this heterogeneity in forecast distributions on the eve of the recent financial crisis, survey respondents' mean forecasts showed an interquartile range

[7]This measure equals zero for any two identical distributions and is bounded above by 1 (for two disjoint distributions).

[8]An alternative would be to define the contribution of heterogeneous means by evaluating (**??**) at identical standard deviations. The resulting contribution of heterogeneous standard deviations to the disagreement measure is somewhat smaller in magnitude, but the upward trend over the sample stronger than that in the center-right panel of Figure **??**.

[11]The sample size of the SPF had shrunk to only 15 forecasters before the survey was taken over by the Federal Reserve Bank of Philadelphia during the course of 1990 and coverage increased to around 45, which we deem too low for studying cross-sectional moments. Disagreement measures in the 1980s were indeed extremely volatile, as shown in (?BroerKero2014), where we had overlooked the small sample sizes during that period.

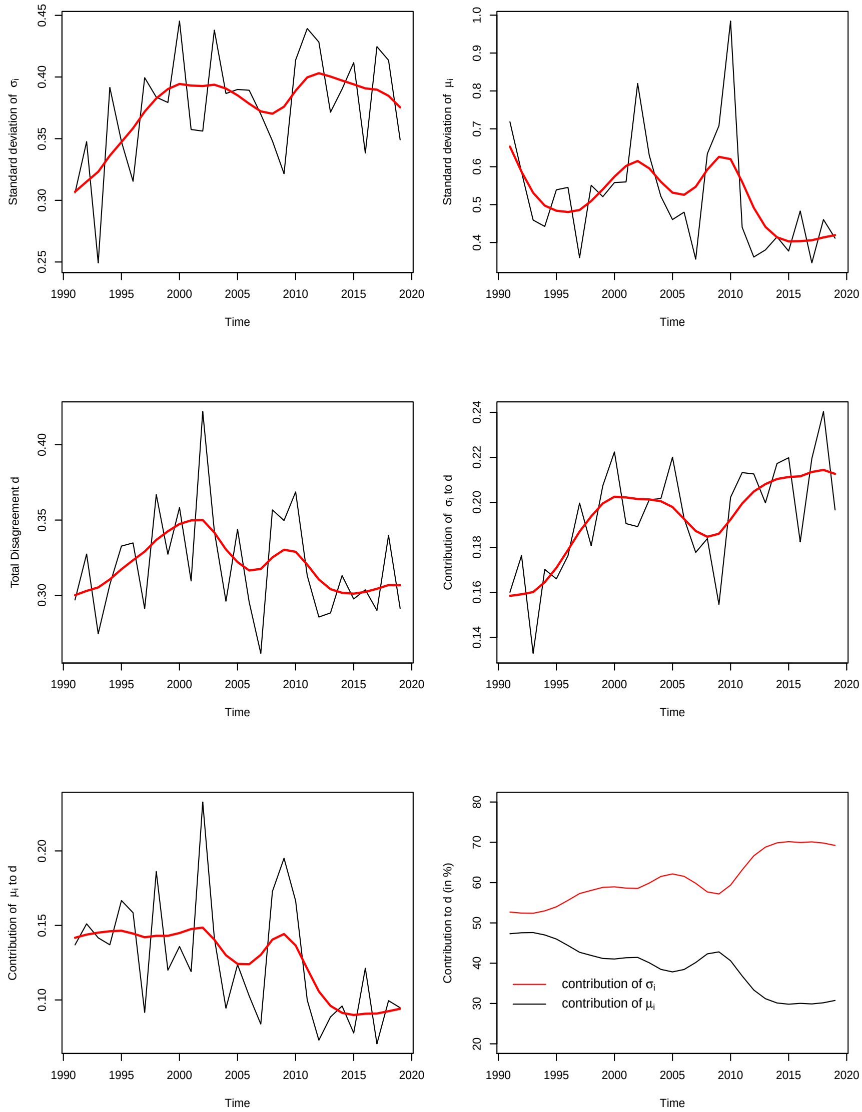


Figure 1: The top-left panel plots the time series of the standard deviations of forecast-standard deviations σ_{it} in the SPF, using a normal approximation to the forecast distribution as in ?GiordaniSoderlind2003 ().[10] The top-right panel plots the standard deviation of means μ_{it}. To remaining four panels show the total disagreement measure d (center-left panel), the contribution of heterogeneous forecast standard deviations σ_{it} (center-right panel), and of heterogeneous forecast means (bottom-left) panels, as well as the percentage of disagreement accounted for by those two parameters (bottom-right panel). The red lines in the first 5 panels show the trend from an HP filter with smoothing parameter 25 (to adjust for the annual frequency, see ?RavnUhlig2002 ()). We omit two observations at the beginning and end of the sample to reflect the two-sided nature of the filter.

of 0.37 percentage points around a median growth forecast of 3.3 percent in the first quarter of 2006. Heterogeneity in the uncertainty around their mean forecast was such that 10 percent of forecast distributions had interquartile ranges of respectively below 0.8 percentage points and above 2.1 percentage points.

While the relative magnitude of the standard deviations in the top row of Figure **??** is difficult to compare, the remaining panels of Figure **??** summarize the overall disagreement measure d in Equation (**??**) (center-left panel) and its decomposition into heterogeneous forecaster-specific means (bottom-left panel) and heterogeneous standard deviations (center-right panel). Overall forecaster disagreement fluctuated around a roughly inverse-U shaped pattern that masks strongly opposing contributions from disagreement about standard deviations (which rose strongly during the second half of the 1990s and again after 2007, by a total of about 1/3) and about mean growth (which falls by about 1/3 over the sample). The bottom-right panel shows how the percentage of overall disagreement that can be attributed to disagreement about forecast dispersion is substantial and rises throughout most of the sample period. Disagreement about mean growth has conversely become less important.

2.3 Heterogeneous risk perception and correlation trades

We believe that the evidence about heterogeneous perceptions of macro volatility in the previous section is interesting for at least two reasons. First, fluctuations in aggregate output growth are an important determinant of asset payoffs in many contexts. And second, it is precisely this disagreement about the (relative) importance of macro sources of volatility, as opposed to diversifiable sources at the creditor or regional level, that translates to disagreement about the co-movement of default events when collateral pools consist of more than one asset: the more volatile an investor thinks aggregate factors are relative to idiosyncratic ones, the more correlated she expects defaults to be. In other words, disagreement about macro volatility translates to disagreement about default correlation among loans in a collateral pool.

Building on this insight, there is an alternative source of evidence for disagreement about risk from financial trading patterns called “correlation trades”, an investment strategy that was popular before the post-2007 financial crisis. In such trades, speculators buy credit default swaps referencing CDO tranches in anticipation of their default, but offset the negative cash-flow effect of insurance premia with the returns on long positions in other tranches of the same (or an equivalent) securitisation. Depending on the perceived correlation of defaults in the pool of collateral assets, different kinds of such long-short trades are profitable. For example, investors who perceive low default correlation regard the payoff of collateral assets as little dispersed about its mean. They thus purchase senior tranches (which they perceive as riskless) at the same time as insurance against the default of riskier ones (which they perceive to be junk). Investors who perceive volatile aggregate conditions, implying high correlation and volatile default rates, in contrast, find it profitable to buy protection against senior losses (which they perceive to be likely) and to insure, or buy, risky tranches (whom they perceive to have upside potential). The former trading strategy was pursued, for example, by Morgan Stanley’s Global Proprietary Credit Group, which suffered one of the biggest (and through Michael Lewis’ bestseller “The Big Short” (?Lewis2010) best-known) trading losses in financial history as the subprime crisis unfolded. The ‘Magnetar Trade’, in contrast, was hugely profitable for the US hedge fund of the same name that simultaneously invested in CDO equity tranches and in credit default swaps for more senior tranches (?maehlmann2011). Indeed, these correlation trades are almost a unique feature of the structural finance boom of the mid-2000s. With little trade before the early 2000s, volume surged in 2004 and 2005 (Corb (2012), p. 415), before falling back to insignificance after the financial crisis hit. Already in May 2005, however, hedge funds had experienced large losses from correlation trades in corporate CDOs, after buying equity tranches and selling protection on more senior tranches in anticipation that credit movements would be broadly correlated. When both GM and Ford saw their credit rating downgraded to junk-status in May 2005, the strategy implied large losses as equity tranches lost value but more senior tranches moved little in response to what was perceived as an idiosyncratic downgrade. The increasing popularity of correlation trades is partly due to financial innovation that made such strategies easier to pursue. It is nevertheless an indication that some investors perceived a substantially different correlation of downgrades and defaults, and therefore a different riskiness of credit collateral, than the market on average.

Our conclusion from the evidence presented in this section is twofold. First, perhaps unsurprisingly, there is strong disagreement in US surveys about the dispersion of asset returns and macroeconomic outcomes that contributes to our overall disagreement measure a similar amount as more commonly documented heterogeneous means. Second, disagreement about GDP growth risk in the SPF increases in the 1990s and 2000s both in absolute terms and, in particular, relative to disagreement about mean growth. Since this increase in disagreement coincides with the period where economists first identified the Great Moderation in macro-volatility (?McConnell97Output; ?kim1999has), one possible explanation for it are diverging opinions about the origin of the Great Moderation in, for example, 'good luck', 'good policy' or changes in the structure of developed economies, with contasting implications for its persistence. The timing of this rise in disagreement about payoff dispersion also suggests it as an additional factor that encouraged issuance of debt securities backed by risky collateral from the early 2000s onwards.[12]

3 Theory: How disagreement about risk can lead to a bubble in collateral prices

This section studies a simple equilibrium economy where two risk-neutral investor types disagree about the riskiness, but not the expected value, of payoffs from an asset (or a pool of collateral loans) that they can buy and use as collateral for issuing debt. We show two results: first, self-selection of investors into holding, respectively, a senior debt tranche and a junior equity tranche raises the equilibrium price of the mortgage pool above its common expected payoff. Second, we derive conditions for an increase in disagreement to increase the equilibrium price

[12]See also footnotes ?? and ?? above.

further. We also compare our main results to the case of disagreement about mean payoffs.[13]

3.1 The general environment

We study an economy that exists for two periods $t \in \{0, 1\}$, with two types of agents $i \in \{H, L\}$, both of unit mass. In period 0, agents of type i receive an endowment $n_i > 0$ of the unique perishable consumption good and 1 unit of a risky portfolio that contains a large number of risky assets, indexed by l. We call these assets 'mortgages', but they could be any other kind of risky claims. Mortgages pay an exogenous stochastic amount s_l in period 1 that is bounded by a recovery value V_{rec} below, and by their face value 1 above. For concreteness, assume $s_l = E^s + (1 - \theta_i)\varepsilon_l + \theta_i\varepsilon$ where $E^s < 1$ is a positive constant, ε_l a loan-specific and ε a common random shock. ε_l and ε follow independent and continuous distributions on a finite support $\mathbb{E} = [V_{rec} - E^s, 1 - E^s]$ with 0 mean. In particular, the distribution of ε is described by a cumulative distribution function F_ε that is continuous and strictly increasing on its support $[\underline{\varepsilon}, \bar{\varepsilon}] \subseteq \mathbb{E}$. Assuming that idiosyncratic shocks $\varepsilon_l, \varepsilon_k, \forall l \neq k$ are independent the mortgage portfolio pays $s = \int s_l dl = E^s + \theta_i\varepsilon$ by the law of large numbers.

Agents agree about mean payoff E^s but disagree about the relative importance of aggregate risk for mortgage payoffs. For this, we normalise the distribution of ε to be common across investors, but allow heterogeneity in perceptions of θ_i, which can be interpreted as the perceived relative variability of aggregate vs idiosyncratic determinants of loan defaults, and determines the perceived variance of payoffs from the mortgage pool. Specifically, we assume $\theta_L < \theta_H$: the 'high-risk' type H believes that the payoff variance of the mortgage portfolio is higher, and payoffs thus less tightly distributed, than the 'low-risk' type L.

We denote the cumulative distribution function of s perceived by type i as $F_i : S \longrightarrow R^+$,

[13] The focus on trade in two assets, senior debt and junior equity, is without loss of generalisation in our simple setting with only two investor types (whose perceived cash-flow distributions we assume only cross once). Section **??** considers more complex assets with several investor types, and the Online Appendix looks at extensions to risk aversion, and trade in additional assets such as riskless storage and options. A previous working paper version of this article (?BroerKero2014) presents results for the general case with a continuum of types.

with f_i the corresponding pdf. Our assumptions imply that F_L strictly second order dominates F_H, which it crosses exactly once at $\varepsilon = 0$. The main results in this section continue to hold under the more general assumption that beliefs about payoffs satisfy second order stochastic dominance with a common mean.

In our prefered interpretation of this environment, heterogeneous risk perceptions arise from contrasting views about the importance of aggregate vs. idiosyncratic factors in determining mortgage defaults. Investors who think that loan-specific factors are the dominant source of defaults, and thus think that diversification through pooling can eliminate most risk, expect the pool's payoff to be tightly distributed around its mean and regard it as high-quality collateral for debt. At the same time they see little upside potential in a leveraged pool's payoff after the debt it collateralizes has been paid. Those, in contrast, who think loan risk comes mostly from aggregate shocks, or who perceive these to be more volatile, believe in volatile payoffs. They thus expect collateralized debt to default in bad times but equity tranches to pay when times are good. Self-selection of investors then raises the prices of both collateralized debt and of the collateral pool. An equally valid interpretation of our framework, however, is one where investors leverage purchases of a single risky asset using collateralized debt.

All agents are risk-neutral with preferences $U_i = c_i + \frac{1}{R}E_i(c_i')$, where E_i is the mathematical expectation of agent i, c_i (resp. c_i') denote consumption in period 0 (resp. 1) and $\frac{1}{R} \leq 1$ is the discount factor. At the end of $t = 0$ agents trade the mortgage pool at unit price p. While we assume agents cannot trade uncollateralized claims (for example because there is no commitment to repayments in the final period 1) they can use the cash flow from the mortgage pool as collateral for structured debt securities. We concentrate on the simplest form of these securities, which allocate the cash flow from the mortgage pool to a debt and equity tranche, but consider more realistic, complex structures in Section **??**. The debt tranche is a senior claim on the collateral cash flow that promises to pay a face value $\overline{s}$ or the payoff of the loans that serve as collateral, whatever is smaller. Normalizing contracts to have 1 unit of the portfolio as collateral[14], the debt tranche thus has unit payoffs equal to $\min\{s, \overline{s}\}$, which are trivially

[14]Note that one unit of the debt tranche collateralized by x units of the pool is payoff-equivalent to x

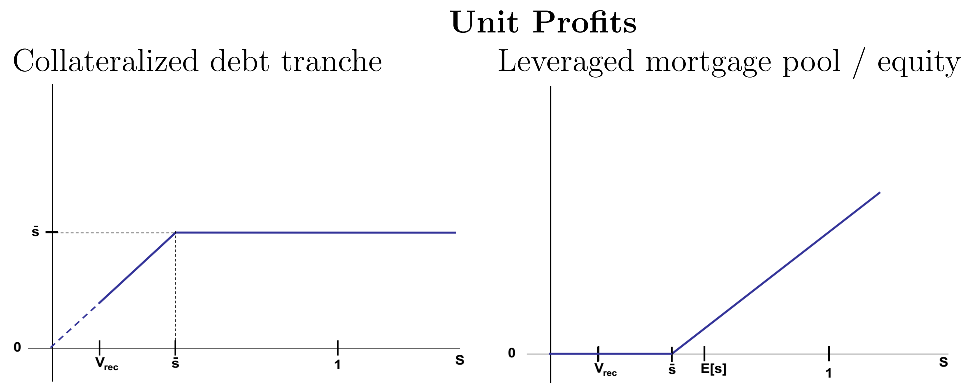


Figure 2: The left panel plots the profits from collateralized debt that are concave in s and the right panel plots those from purchases of the equity tranche, or from leveraging the mortgage portfolio by issuing collateralized debt, which are convex in s.

concave in s. The equity tranche simply pays the remainder $\max\{0, s - \overline{s}\}$, which is convex in s. Because buying the mortgage pool and selling - or issuing - the debt tranche is payoff-equivalent to buying the equity tranche, there is an obvious multiplicity in portfolios. In the following, we therefore concentrate on trade in the mortgage portfolio, possibly leveraged by issuing debt tranches that trade at price $q(\bar{s})$, which we call 'collateralized debt' for simplicity. These debt contracts must fulfill the collateral constraint

$$\int_0^1 b_i(\bar{s}) d\bar{s} \geq -a_i \tag{2}$$

where $b_i(\bar{s})$ are agent i's holdings of collateralized debt contracts with face value $\bar{s}$, and $a_i > 0$ denotes her holdings of the mortgage pool.

The budget constraints of agent i in $t = 0$ and $t = 1$ respectively are:

$$c_i + pa_i + \int_0^1 q(\bar{s}) b_i(\bar{s}) d\bar{s} \leq n_i + p, \tag{3}$$

$$c_i' \leq a_i s + \int_0^1 min\{s, \bar{s}\} b_i(\bar{s}) d\bar{s}, \tag{4}$$

Figure **??** illustrates the (gross) unit profits of different investments as a function of the mortgage

units of a tranche with face value 1/x collateralized by one unit of the pool.

pool's payoff s, for given face value $\bar{s}$. While profits from a unit of collateralized debt (left panel), equal to $\frac{[min\{s,\bar{s}\}]}{R}$, are concave in s, those from a mortgage portfolio fully leveraged by issuing an equal amount of units of debt with face value $\bar{s}$ (right panel) are $\frac{s-\min(s,\bar{s}))}{R} = \frac{\max(0,s-\bar{s}))}{R}$ and thus convex in s. Given the second order stochastic dominance relationship of beliefs, this immediately implies that investors with more dispersed beliefs expect to make higher profits from investment in the leveraged mortgage portfolio, denoted as Π_i^a. Investors with less dispersed beliefs expect higher profits from debt tranches Π_i^d.

Lemma 1 - *Profits and risk perceptions*

Consider a given face value of debt contracts $\bar{s}$. Type L investors expect higher profits from investing in risky collateralized debt than type H investors. The inverse is true for profits from the leveraged mortgage portfolio:

$$\Pi_H^d = E_H\left[\frac{[min\{s,\bar{s}\}]}{R}\right] - q(\bar{s}) \leq \Pi_L^d,\ \forall\ \bar{s} \in (V_{rec},1), \forall p, q(\bar{s}), R,$$

$$\Pi_H^a = E_H\left[\frac{\max(0,s-\bar{s}))}{R} - p + q(\bar{s})\right] \geq \Pi_L^a,\ \forall\ \bar{s} \in (V_{rec},1), \forall p, q(\bar{s}), R.$$

Moreover, there exists $\bar{s} \in (V_{rec},1)$ such that both equalities are strict.

The proof of Lemma **??** follows immediately from the strict concavity (convexity) of Π_i^d (Π_i^a) at $\bar{s} \in (V_{rec},1)$, and the strict second-order stochastic dominance relationship of beliefs. It follows that type L agents are the natural buyers of collateralized debt, and H agents are the natural investors in the leveraged mortgage portfolio. In other words, if there is trade in collateralized debt in equilibrium $-b_H = b_L > 0$.

3.2 Equilibrium definition

Definition 1 *A general equilibrium is a set of prices $(p,\, q(\bar{s}))$ and allocations $\{c_i, c_i', a_i, b_i(\bar{s})\}_{i\in\{L,H\}}$ $\forall \bar{s}$, such that both agents optimally choose their consumption and investments subject to their budget constraint and the collateral constraint* (**??**)*, the demand for the mortgage portfolio equals* *the fixed supply, and the collateralized debt market clears,*

$$b_H(\bar{s}) + b_L(\bar{s}) = 0, \ \forall \bar{s}.$$

3.3 Equilibrium characterization

Despite the simple nature of the environment, the equilibrium of the economy is complex because portfolios may include long and short positions in a continuum of debt contracts indexed by their face value $\bar{s}$. To simplify the equilibrium, and to capture the strong demand for senior tranches of RMBS and other securitizations before the post-2007 financial crisis, we assume that type L agents, who are the natural buyers of collateralized debt tranches, have a sufficiently high consumption endowment. We show how this assumption implies a pricing function of collateralized debt that substantially simplifies the characterization of equilibrium. Also, in what follows we normalize the discount factor such that $R = 1$.

Assumption A1 $n_L \geq E^s$.

Assumption **??** has two implications: first, the equilibrium price of a unit of the mortgage portfolio p is bounded below by the fundamental value E^s, since any lower price contradicts goods-market clearing in period 0, as it would give both types at least one investment possibility that they would strictly prefer over current consumption. Second, the total value of type L agents' endowment equals $n_L + p \geq 2E^s \geq 2max_{\bar{s}}\left[E_L[min\{s, \bar{s}\}]\right]$. So type L agents can afford to buy all collateralized debt at its maximum expected payoff. Since, moreover, they do not expect to make strictly positive profits from any other investment, they bid up the price of any collateralized debt issued by type H agents to their expected discounted value, where they are indifferent between investing and consuming, implying a debt price function

$$q(\bar{s}) = E_L[\min\{s, \bar{s}\}]. \tag{5}$$

As stated in Corollary **??**, this implies that type H agents expect profits Π_H from buying the mortgage portfolio outright to be lower than from buying it using leverage, so we can focus on equilibria where type H agents leverage their entire mortgage portfolio holdings without loss of generality.

Corollary 1 *With $q(\overline{s})$ given by (**??**) $\Pi_H^a \geq \Pi_H = E_H[s] - p$, $\forall p, \overline{s}$, with strict inequality for some $\overline{s} \in (V_{rec}, 1)$.*

By requiring that type L agents be able to acquire collateralised debt of *any* face value $\overline{s}$ at their expected payoff, assumption **??** is stronger than what is needed in many cases. In particular, in most equilibria the face value $\overline{s}$ is much below its upper bound, such that smaller type L resources would also suffice to imply the bond-pricing function (**??**) for the relevant range of values of $\overline{s}$.

3.3.1 Type H's problem and the choice of $\overline{s}$

The debt price function (**??**), together with Lemma **??** and Corollary **??**, substantially simplify H investors' portfolio choice problem. This is because they imply that H investors expect to make strictly higher profits from investing in the mortgage pool, once it is optimally leveraged with debt, than from any other investment. Their problem thus simplifies to choosing current consumption (which through the budget constraint determines their investment in the mortgage pool) and the level of leverage $\overline{s}$ given p and the price function $q(\overline{s})$:

$$\max_{c_H, \overline{s}} \quad U_H \quad = \quad c_H + (n_H + p - c_H) R_H^a. \tag{6}$$

where $R_H^a(p, \overline{s}) \doteq \frac{E^s - E_H(\min\{s, \overline{s}\})}{p - E_L\{\min\{s, \overline{s}\}\}}$ is the leveraged gross return of the mortgage pool using debt with riskiness $\overline{s}$.

The first order condition for $\overline{s}$ can be written as:

$$\frac{(n_H + p - c_H)}{p - E_L\{min\{s, \overline{s}\}\}}[(1 - F_H(\overline{s})) - R_H^a(1 - F_L(\overline{s}))] = 0. \tag{7}$$

Equation (**??**) describes the tradeoff between higher debt repayments and increased funds for investment when choosing $\bar{s}$. In particular, a unit-increase in $\bar{s}$ increases H investors' expected discounted repayments by $1 - F_H(\bar{s})$, proportional to their perceived probability of paying back the full face value. It increases expected returns by the increase in the price of collateralized debt (equal to $(1 - F_L(\bar{s}))$) multiplied by the expected return on investment R_H^a. Importantly, to the right of the single-crossing point E^s, H investors perceive a higher probability of full repayment than L investors ($1 - F_H(\bar{s}) \geq 1 - F_l(\bar{s}), \forall \bar{s} > E^s$). A small increase in $\bar{s}$ thus increases H's expected discounted payments by more than the additional funds it raises from L investors. Nevertheless, according to (**??**), whenever H investors perceive the profitability of investing in the leveraged mortgage pool R_H^a to be higher than R, they find it optimal to raise $\bar{s}$ above E^s to raise additional funds for investment. Proposition **??** shows that if debt issuance is profitable for H investors at some face value $\bar{s}$, this tradeoff has an interior solution where (**??**) equals 0. For this, we define H's maximum willingness to pay for a unit of the mortgage pool leveraged with collateralized debt of face value $\bar{s}$, as

$$\overline{p}(\bar{s}) \doteq E^s + E_L(min(s, \bar{s})) - E_H(min\{s, \bar{s}\}) \tag{8}$$

$\overline{p}(\bar{s})$ is the price at which H investors are exactly indifferent between consuming in $t = 0$ and buying a unit of the mortgage pool and leveraging it through debt with face value $\bar{s}$ that raises resources $q(\bar{s})$.

Proposition 1 - *Interior choice of $\overline{s}$.*

Suppose there exists $\overline{s} \in (V_{rec}, 1)$ such that H investors are willing to buy the mortgage pool at a given price p, i.e.

$$\exists \overline{s} \in (V_{rec}, 1) : p \leq \overline{p}(\bar{s}). \tag{9}$$

Then $R_H^a(p, \bar{s})$ has an interior maximum at some $\overline{s}^\star \in (V_{rec}, 1)$.

Proof of Proposition ??.

Note that p is bounded below by E^s from assumption **??**. Also, $R_H^a(E^s, 1) = 1$ by a limit argument. If $p = E^s$, then $R_H^a(E^s, V_{rec}) = R_H^a(E^s, 1) = 1$ and condition (**??**) holds with strict inequality for $\overline{s} = E^s$, implying $R_H^a(E^s, E^s) > 1$. If $p > E^s$ then $R_H^a(p, V_{rec}) < 1$ and $R_H^a(p, 1) < 1$. (**??**) implies that for some $\overline{s}' \in (V_{rec}, 1)$ $R_H^a(p, \overline{s}') \geq 1$. In both cases the statement then follows from continuity of R_H^a. ■

3.3.2 The equilibrium price of mortgage collateral

Proposition **??**, whose proof is in the appendix, shows that the equilibrium in this economy is defined by two conditions: first, the optimal choice of leverage $\overline{s}$; and second, asset market clearing for the mortgage pool, which defines the price such that H investors either exhaust all their wealth buying the entire mortgage pool, or are indifferent between investing and consuming. Intuitively, as wealth of H investors rises, their increasing demand for the mortgage portfolio bids up the price until it reaches indifference level $\overline{p}$.

Proposition 2 - *Existence and uniqueness of equilibrium.*
Denote as $\mathbb{B}(\overline{s}) = n_H + 2E_L[\min\{s, \overline{s}\}]$ the resources available to H investors for net purchases of mortgage collateral when they issue debt of face value $\overline{s}$ collateralized by the whole mortgage portfolio in the economy. p and $\overline{s}$ are given by the unique solution of the following equations:

$$\mathbb{C} \equiv (E^s - E_H[\min\{s, \overline{s}\}])(1 - F_L(\bar{s})) - (1 - F_H(\bar{s}))(p - E_L[min\{s, \overline{s}\}]) = 0, \quad (10)$$

$$p = min\{\overline{p}(\overline{s}), max\{E^s, \mathbb{B}(\overline{s})\}\}, \quad (11)$$

*The price of collateralized debt $q(\overline{s})$ is given by (**??**). Type H investors purchase the entire mortgage pool and use it as collateral for debt with face value $\overline{s}$. If this does not exhaust their first-period resources, they consume the rest. Similarly, type L agents purchase all collateralized debt, and consume any remaining available resources.*

Equation (**??**) states that the price of the mortgage pool is equal to $\mathbb{B}(\overline{s})$, the funds for investment available to H investors, but bounded by its fundamental value E^s below, and by

H's maximal willingness to pay $\overline{p}(\overline{s})$ above. Note that, from (**??**), when $p = \bar{p}(\bar{s})$, H investors optimally set $\bar{s} = E^s$. This is because at $\bar{p}$, their expected return on investment R^a_H equals that which they have to pay to L investors. There are thus no gains from raising more funds by increasing $\bar{s}$ above the single-crossing point E^s where the difference in expected payments on collateralized debt is maximised.

Proposition **??** immediately implies that whenever type H investors have sufficient resources, there is a bubble in the equilibrium price of mortgage collateral p, defined as a price that exceeds the fundamental expected value of the mortgage pool E^s that is common to both investor types. This is expressed in corollary **??**. For this we say that the underlying, commonly perceived distribution of aggregate shocks ε (on which H and L investors load with heterogeneous factors $\theta_i, i = H, L$) has right-hand skew whenever $F_\varepsilon(0) > \frac{1}{2}$, such that there is more mass to the left of the mean than to its right.

Corollary 2 *A bubble in collateral prices.*[15]
There is a bubble in the price of collateral assets, in the sense that the equilibrium price p strictly exceeds the fundamental value E^s, if one of the following conditions holds

1. *The consumption endowment of H investors satisfies $n_H > E^s - 2E_L[\min\{s, \overline{s}\}]$.*

2. *There is no right-hand skew in F_ε, such that $F_\varepsilon(0) \leq \frac{1}{2}$.*

Proof: Under condition 1, $\mathbb{B}(\overline{s}) > E^s$ and $\bar{p} > E^s$, so (**??**) implies the result. To see how condition 2 also implies $\mathbb{B}(\overline{s}) > E^s$, note that (**??**) implies $(1 - F_L(\bar{s})) \leq (1 - F_H(\bar{s}))$, which holds only if the face value of debt weakly exceeds the single crossing point, such that $\bar{s} \geq E^s$. This implies that the resources from issuing collateralized debt equal $2E_L[\min\{s, \overline{s}\}] \geq 2E_L[\min\{s, E^s\}] \geq 2E^s(1 - F_L(E^s)) \geq 2E^s\frac{1}{2} \geq E^s$, implying $\mathbb{B}(\overline{s}) > E^s$, where the first inequality follows because because $\bar{s} \geq E^s$ and $E_L[\min\{s, x\}]$ is increasing in x, the second

[15] A previous working paper version of this article, ?BroerKero2014 (), shows that similarly to the two-type economy, with heterogeneity in perceived risks across a continuum of types, the equilibrium price of the loan portfolio are necessarily above their common fundamental valuation. Unlike the two-type economy, however, these results require an exogenous upper bound for the face value $\bar{s}$.

inequality follows because payments on collateralized debt in states of full repayment are weakly smaller than total payments, and the third inequality imposes condition 2.

To understand the intuition behind condition 2, note that for given expected payoff E^s, an increase in right-hand skew moves more probability mass below E^s (or probability mass below E^s to the left). This necessarily decreases the pay-offs of collateralised debt issued at face value $\overline{s} = E^s$. When condition 2 holds, in contrast, the proceeds from issuing collateral debt at the optimal face value always suffice to drive the price of mortgage collateral above its fundamental value. So disagreement implies a bubble in collateral prices even when H investors have no own funds for investment because their consumption endowment equals 0. Conversely, Condition 2 implies Condition 1 whenever $n_H > 0$. Moreover, whenever the equilibrium price is strictly between the fundamental value and its upper bound $\bar{p}$, any increase in H's endowment n_H drives up prices. This is stated in Corollary **??**.

Corollary 3 *An increase in H's resources inflates the bubble.*
A rise in the endowment of type H investors n_H strictly increases the equilibrium price of collateral p when $\mathbb{B}(\overline{s}(E^s)) > E^s$ and $\mathbb{B}(\overline{s}(\bar{p})) < \bar{p}$.

3.4 Belief divergence and collateral prices

This sub-section looks at the effect of 'belief divergence', defined as a further increase in the difference between θ_L and θ_H through infinitesimal changes $d\theta_L < 0$ and $d\theta_H > 0$.[16] By increasing the likelihood of high payoffs, $d\theta_H > 0$ increases type H's perceived upside potential of leveraged payoffs from the pool. And $d\theta_L < 0$ further reduces the riskiness of the collateralized debt as perceived by L types, and thus increases the price $q(\overline{s})$. For a given face value of the debt tranche $\overline{s}$, belief divergence thus raises type H's expected return on the leveraged mortgage portfolio, as well as her resources from debt issuance, which increases the equilibrium price of the mortgage pool p. According to (**??**) this is, however, complicated by adjustment in $\overline{s}$ whenever

[16] Again, the results in this section also hold with a general mean-preserving spread (contraction) in type H (L)'s beliefs about a generic stochastic payoff s. The results are available from the authors.

$\bar{s} > E^s$: $d\theta_H > 0$ then increases the probability of full repayment $1-F_H(\bar{s})$, equal to the marginal cost of an increase in $\bar{s}$. This gives H types incentives to reduce $\bar{s}$ in response to an increase in perceived upside risk, which may reduce the general equilibrium collateral price. Similarly, when $\bar{s} > E^s$ a reduction in payoff risk perceived by low types increases their perceived probability of default $1 - F_L(\bar{s})$, giving H types an additional incentive to reduce the face value of debt. The following proposition, whose proof is in an appendix, identifies sufficient conditions for belief divergence to raise the price of the mortgage pool.

Proposition 3 - *Belief divergence $d\theta_L < 0, d\theta_H > 0$ increases p if any of the following conditions hold*

1. *The distribution of ε is not right-skewed ($F_\varepsilon(0) \leq \frac{1}{2}$).*
2. *Endowments of high types exceed a threshold $n_H > \underline{n_H} = E^s - E_L(min\{s, E^s\}) - E_H(min\{s, E^s\})$.*
3. *Both types perceive a zero probability of $s = \bar{s}$, so belief divergence leaves the default probability perceived by either type unchanged.*

Conditions 1 and 2 imply that H types can afford to buy the whole mortgage pool of the economy at $p = \overline{p}$. This implies $\bar{s}$ equals E^s and is unaffected by belief divergence. There is thus no fall in $\bar{s}$ that could act to reduce p. Type H resources can be high either because the probability of full repayment $1 - F_L(\bar{s})$ perceived by the low types, and therefore the proceeds from issuing collateralized debt, are high (condition 1)[17]; or because high types have a large enough consumption endowment (condition 2). Condition 3, in contrast, implies that belief divergence leaves the default probability at the original level of riskiness $\bar{s}$ unaffected. According to (**??**), H investors thus optimally increase $\bar{s}$ to raise more funds from L agents as belief divergence raises their expected return on investment R_H^a.

Importantly, these conditions are sufficient but not necessary: as long as the probability of full repayment is sufficiently flat around $\bar{s}$ (and $\frac{1-F_i(\bar{s})}{d\theta_i}$ thus small), belief divergence still

[17] Proposition 6 in `?Ellisetal17` () makes a similar argument for a symmetric distribution.

increases the prices of debt and of the mortgage pool. It is, however, easy to construct counterexamples where $\frac{1-F_L(\bar{s})}{d\theta_L}$ is large enough to make belief divergence decrease prices. In particular, when L investors perceive payoff $\hat{s} > E^s$ to have high probability mass, such that F_L increases discretely at $\hat{s}$, H investors may find it optimal to choose face value $\bar{s} = \hat{s}$. Whenever a reduction in risk perceived by L investors shifts the probability mass below $\hat{s}$, the equilibrium price of debt may fall sufficiently much to decrease equilibrium collateral prices, as the following example shows.

Example 1 *Belief divergence may decrease asset prices*
For $V_{rec} = 0$, consider the limit case of a discrete distribution on support $\{0+a_i, 0.25+b_i, 0.75-b_i, 1-a_i\}, i \in \{H, L\}$ with probability masses $p_H = \{0.5, 0, 0, 0.5\}$ and $p_L = \{0.25, 0.25, 0.25, 0.25\}$. Take $R = 1$, $n_L = 0.5$, $n_H = 0.2$. It is trivial to show that type H sets $\bar{s} = 0.75 - b_L$ for small enough b_L as the profit function has a kink at that value: raising $\bar{s}$ above $0.75 - b_L$ would increase her expected payments by more than the expected gain from higher price of collateralized debt, $\bar{s} < 0.75 - b_L$ would leave cheap debt unused. This implies $q = 0.4375$ and $p = \bar{p} = 0.5625$ when $a_i = b_i = 0$. Consider an increase in disagreement in form of small increases in either b_L or a_L. Since $\frac{dp}{db_L} = -0.25$ and $\frac{dp}{da_L} = 0.25$, belief divergence may increase or decrease the price of the loan portfolio.

3.5 Comparison with disagreement about mean payoffs

This section briefly illustrates the main differences of our benchmark results with respect to the case where there are optimist and pessimist investors who disagree about mean payoffs of the collateral pool, in the sense that the payoff distribution perceived by optimists dominates that of pessimists at first order (rather at second order as in our benchmark analysis), as analyzed by ?Simsek2013 (). Optimists expect payoffs from all (increasing) assets to be higher than pessimists. The collateral price thus crucially depends on optimists' resources, and is bounded by their optimistic valuation. In other words, there is never a bubble in the collateral price as in our benchmark analysis. Rather, when disagreement is concentrated on downside risk, the equilibrium asset price may equal the pessimistic valuation both with and without collateralization (as pointed out by ?Simsek2013 ()).

Consider a version of the environment with two alternative investor types that are pessimists and optimists, denoted P and O, respectively. Investors agree about the recovery value of the mortgage portfolio V_{rec}, but disagree about the mean payoff, in the sense that the distribution of payoffs perceived by optimists first-order stochastically dominates that perceived by pessimists, with strict inequality for all payoffs above the recovery value, such that $1-F_O(s) > 1-F_P(s) \forall s \in (V_{rec}, 1)$.

The following corollary exploits the fact that, under assumption **??**, the bond price function (**??**) is unchanged.

Corollary 4 *No bubble in collateral prices with disagreement about mean payoffs.*
With disagreement about mean payoffs, there is no bubble in collateral prices. Moreover, the equilibrium price of collateral p is strictly lower than the optimistic valuation E_O^s when the recovery value V_{rec} or optimist resources are sufficiently low, such that $n_O < E_O^s - 2V_{rec}$ and optimists cannot fund the purchase of the mortgage portfolio using only own funds and riskless debt.

Proof. Note that the maximum price investors are willing to pay when issuing collateralized debt of face value $\bar{s}$ still equals $\overline{p}(\bar{s})$ in (**??**), with changed subscripts

$$\overline{p}(\bar{s}) \doteq E_O^s + E_P(min(s, \bar{s})) - E_O(min\{s, \bar{s}\}) \tag{12}$$

First-order stochastic dominance implies $E_P(min(s, \bar{s})) - E_O(min\{s, \bar{s}\}) \leq 0$, with strict inequality for $s \in (V_{rec}, 1]$, since the payments on collateralized loans that optimists expect to make are larger than the receipts expected by pessimists. Thus $\overline{p}(\bar{s}) \leq E_O^s$: the collateral price is bounded above by the optimist valuation. Whenever the sum of optimist endowment n_0 and funds raised by issuing riskless debt at $\bar{s} = V_{rec}$ does not suffice to buy the whole mortgage portfolio at $p = E_O^s$, the market price of collateral is bounded above by $max_{\bar{s} \in (V_{rec}, 1]} \overline{p}(\bar{s}) < E_O^s$.

■

The standard case of disagreement about mean payoffs thus does not imply a bubble in collateral prices, which remain bounded by the valuations of individual investors. But the possibility to use the asset as collateral for debt may increase its price by putting more funds into the hands of optimists.

Corollary 5 *Positive return to collateralization with disagreement about mean payoffs*
With disagreement about mean payoffs, the collateral price p may exceed the value of the mortgage portfolio sold without collateralization. In other words, there may be a positive return to collateralization.

The proof is by example.

Example 2 *Positive return to collateralization with disagreement about mean payoffs*
Consider $E_P = V_{rec}$, $E_O = 1$, $n_P = V_{rec}$, and $1 - 2V_{rec} < n_O < n_P$. In this case, it is easy to see that the market price of collateral when collateralized loans are not traded equals V_{rec}, the pessimist valuation (since optimists cannot purchase pessimists' mortgage portfolio at any price higher than that). With collateralized loan trade, in contrast, optimists optimally buy pessimists' mortgage portfolio at a price of 1, equal to their own optimistic valuation, which they can afford by issuing loans at the optimal face value $\bar{s} = V_{rec}$.

Note the difference to our benchmark analysis: with disagreement about risk, a positive return to collateralization requires a bubble in asset prices. With disagreement about mean payoffs it does not. A final example shows how the return to collateralization may be zero, and the collateral price equal to the pessimist valuation, when disagreement is concentrated on downside risks.[18] We choose a particularly stark example where optimists and pessimists only disagree about the probability of catastrophic losses (where payoffs are zero).

[18]See ?Simsek2013 () for a more general discussion of disagreement about upside vs. downside risk.

Example 3 *No return to collateralization with disagreement about downside risk*
Consider a case where optimists and pessimists agree on the payoff distribution, but pessimists perceive a small probability $\mathbb{p}$ *of a catastrophic loss (where the mortgage pool pays zero), while optimists perceive no possibility of such loss. Assume that optimists lack funds to buy the whole asset endowment of pessimists at the minimum price, equal to the pessimistic valuation of collateral cash-flow* E_P^s*. Because* $E_P(min(s, \bar{s})) - E_O(min\{s, \bar{s}\}) = E_P^s - E_O^s$ *in* (**??**)*, the price of the collateral is unaffected by collateralization and equal to the pessimistic valuation.*

Here, because disagreement is about catastrophic losses, and thus concentrated at the bottom of the payoff distribution, optimists do not gain from issuing debt even when the price of the asset is low (equal to E_P^s): the expected losses they make from issuing debt at the pessimist valuation exactly equal the gains they expect to make from buying the asset at the low price.

4 Quantification: The effect of disagreement about risk on the Structured Finance boom

This section studies the quantitative effect of disagreement about risk on the price of realistic structured debt securities namely the residential mortgage backed securities (RMBSs) backed by their tranches that experienced an unprecedented boom before the recent global financial crisis. We consider a version of the economy in Section **??** with $i = 1, .., I$ types of risk-neutral investors and homogeneous consumption endowment n. Collateral assets consist of a pool of $l = 1, ..., L$ mortgages of face value and mass 1. To keep the analysis tractable with many investors, we assume that the whole mortgage pool is endowed to a single originator who sells it in its entirety to investors in $t = 0$ in one of two ways: as exogenously given tranches or as simple shares as explained below,.[19]

In $t = 1$ a stochastic fraction d of mortgages defaults and pays recovery value $V_{rec} < 1$. We

[19] After this paper was complete, ?Ellisetal17 () considered the optimal security design in a similar framework.

exploit the fact that before the crisis there was a unique "market standard' (?morini2011 (), p. 127) model of credit risk, the standard Gaussian copula model with homogeneous correlation (?li2000; ?laurent2005). We assume investor i believes in this model, and thus expects mortgage l to default whenever the following condition is met

$$x_l = \rho_i \cdot M + \sqrt{1-\rho_i^2} \cdot M_l < \overline{x} = \mathbb{N}^{-1}(\overline{\pi}), \ M, M_l \sim N(0,1) \tag{13}$$

The index variable x_l can be interpreted as the value of creditor l's assets. It equals the weighted average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_l, which are both distributed according to the standard normal distribution. Investors agree that loan l defaults whenever the index x_l falls below a threshold $\overline{x}$ equal to the inverse normal distribution evaluated at the default probability $\overline{\pi}$, which this section assumes is shared by all investors. As before investors disagree about the importance of aggregate conditions in determining loan defaults, as summarized by the parameter ρ_i. In this version of the model, ρ_i^2 equals the correlation between two individual creditors' asset values perceived by investor i. Again, investors with higher perceived ρ_i believe individual defaults to comove more strongly, and thus expect d to be less tightly distributed around $\overline{\pi}$. The normalization of the aggregate factor M to unit-variance implies that any disagreement about the volatility of aggregate conditions will be captured by the parameter ρ_i. The disagreement about macroeconomic risk among forecasters in Section **??** is thus suggestive evidence for heterogeneous values of ρ_i. Together with the recovery value in case of default V_{rec}, ρ_i and $\overline{\pi}$ completely determine the distribution of the cash flow from the mortgage pool equal to $s = 1 - d(1 - V_{rec}) \ \forall d$.

The originator maximizes current profits from selling the loan pool to investors in one of two ways: as shares in a 'pass-through' securitization that pays all investors their share in the total cash flow that the collateral generates, equal to $1 - d(1 - V_{rec}) \ \forall d$; or structured as an RMBS by splitting the cash flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Specifically, tranche 1 promises to make a total payment of $a_1 < 1$ to its holders in period 2, where a_1 is the 'detachment point' of tranche 1, and receives any cash flow that defaulting and non-defaulting mortgages generate until a total of a_1 is reached. Tranche 2 promises to pay $a_2 - a_1$, where $a_1 < a_2 < 1$, but only receives cash flow once a_1 has been paid to holders of the first tranche, etc.

Given one of the two securitization possibilities - structured or pass-through - an equilibrium is defined as a vector of prices such that the originator maximizes current profits, investors maximize utility, and demand for all assets equals supply.

4.1 Payoff distributions and valuation of tranches

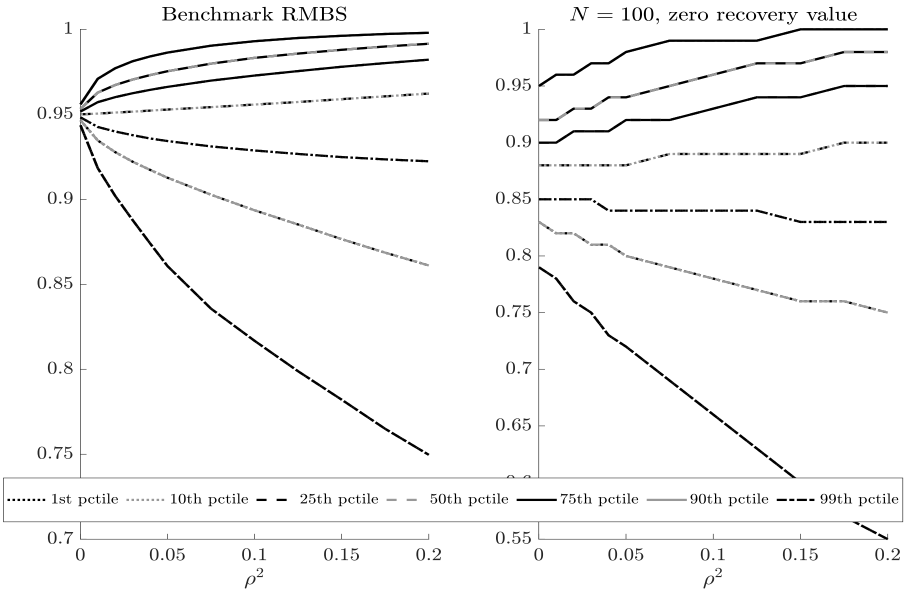
To illustrate how investors' perceived correlation determines the perceived payoff distribution, we choose parameters to capture the characteristics of the market for US subprime mortgage-backed securities prior to the crisis. Specifically, we consider RMBS consisting of 5000 mortgages (?coval2009). We set the common perceived default probability $\overline{\pi}$ equal to 12.5 percent, and a recovery value V_{rec} that comoves inversely with the pool's default rate d in a range of $+/-15$ percentage points around $\overline{V}_{rec} = 60$ percent, reflecting longer time-until-foreclosure and lower resale values when default rates are high.[20] The analysis uses the same 6 tranche structure as ?coval2009 (): equity (100-97 percent), junior (97-93 percent), mezzanine I and II (93-88 and 88-80 percent respectively) and senior I and II (80 to 65 and 65 to 0 percent).

The left panel of Figure **??** shows how, for $\rho_i^2 = 0$, the perceived distribution of payoffs from an RMBS's collateral pool collapses around the expected payoff equal to $1 - \overline{\pi}(1 - \overline{V}_{rec}) = 95$ percent. As ρ_i^2 rises, the distribution fans out at a decreasing rate, but the lowest percentile remains above 75 percent as payoffs are protected by the recovery value.[21] The right panel of Figure **??** illustrates the sensitivity of the perceived payoff distribution to reducing the number of collateral assets to $N = 100$ and the recovery value to 0 (but keeping the default probability of

[20] The default rates for subprime mortgages differed strongly over time, fluctuating around 10 percent during the years of strong house price growth up to 2006 and increasing to above 40 percent thereafter (see, e.g., ?beltran2013 ()). The recovery value equals $V_{rec} = 0.6 + (d - \overline{d})$, where $\overline{d}$ is the average default rate equal to $\overline{\pi}$, but is bounded by a minimum of 45 percent.

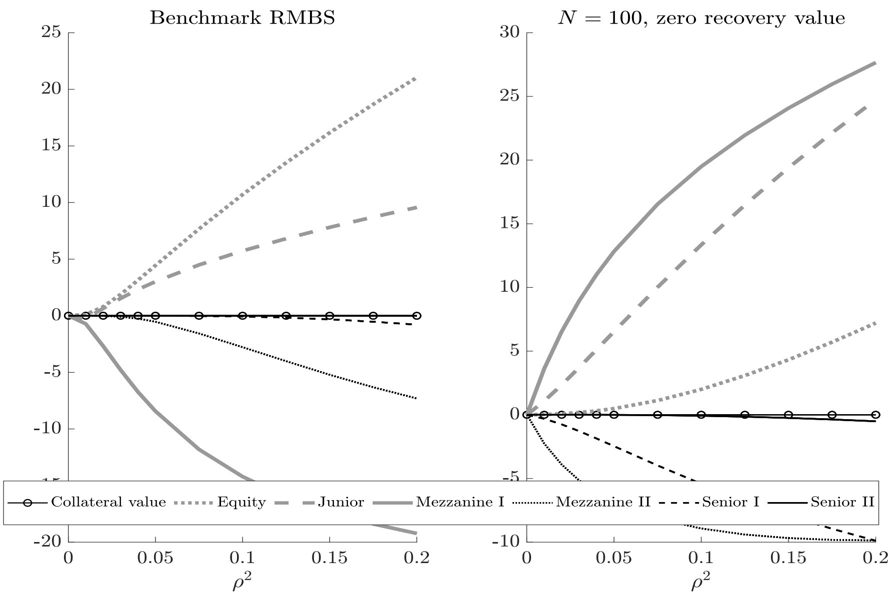
[21] To interpret the magnitudes, note that ρ_i^2 and $\widehat{\rho}_i^2$ do not equal default correlations. In fact, as Figure 3 in ?Broer2018 () shows, the correlation between default events of any two mortgages in the RMBS is about half as large as the correlation of the underlying asset value x_l.

Figure 3: Distribution of collateral payoffs



The figure shows the distribution of collateral payoffs from two pools of mortgages: one with a high number of loans ($N = 5000$) and substantial recovery value (averaging 60 percent, in the left panel); and a second with fewere loans ($N = 100$) and zero recovery value (right panel).

Figure 4: Expected value of tranches



For the two RMBSs in Figure **??**, the figure shows the difference between their tranches' payoffs expected by an investor with perceived asset correlation ρ_i^2 (depicted along the bottom axes) and that expected by a 'zero correlation' investor (whose ρ_i^2 equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

loans equal to 12.5 percent). This makes the asset pool's characteristics more similar to those of a typical CDO consisting of mezzanine RMBS tranches (?coval2009).[22] The payoff distribution in the right panel is markedly different to the benchmark specification: diversification is less powerful with fewer collateral assets, such that even investors who perceive asset payoffs to be uncorrelated ($\rho_i^2 = 0$) perceive cash-flow risk. Moreover, for higher values of perceived correlation, the distribution has substantial mass at values as low as 60 percent, as payoffs are not protected by recovery values, and a positive probability of full repayment.

To illustrate how the heterogeneous perceived payoff distributions depicted in Figure **??** affect the expected payoffs of RMBS tranches, the left panel of Figure **??** shows the difference between their payoffs expected by an investor with perceived asset correlation ρ_i^2 (depicted along the bottom axes) and that expected by a 'zero correlation' investor (whose ρ_i^2 equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche). As expected, the collateral value, or total expected payoff from the mortgage pool (the starred dashed line flat at 0), is unaffected by the perceptions of correlation as all investors share the same average default probability. Because the 'zero correlation' investor expects the payoff from the mortgage pool to equal 95 percent with certainty, she deems the junior and equity tranches of the RMBS, with attachment points close to or above 95 percent, to be worth nothing or little. High ρ_i^2 investors, in contrast, who perceive both a larger downside and upside risk, think that junior tranches are more likely to pay off, while they expect the mezzanine tranches to default with positive probability. Because even investors who perceive high default correlation attach an extremely low probability to default rates of more than 40 percent, and because the recovery value of defaulting mortgage is about 50 percent, investors agree that senior RMBS tranches are (essentially) riskless.

The right panel of Figure **??** shows, again, the sensitivity to reducing the number of collateral assets to $N = 100$ and the recovery value to 0. Cash-flow is thus lower on average and more risky (in the right-hand panel of Figure **??**). This has three implications: first, disagreement

[22] See the working paper version of this article for an analysis of CDOs consisting of junior or mezzanine RMBS tranches, using the same modeling framework.

about expected tranche payoffs is strongest for more senior (here mezzanine) tranches; second, differences in expected payoffs are larger (rising to almost 30 percent for mezzanine I tranches); and finally, these differences in expected values imply higher returns because the value of the more risky collateral pool, when sold as a pass-through securitisation, is lower. Thus, tranching more granular asset pools with higher risk (due to lower recovery values) is potentially more profitable for originators.

4.2 The return to tranching

Structuring a loan pool is profitable for originators whenever they can sell different tranches, but not the pool as a whole, to investors with high valuations. As Figure **??** shows, disagreement about default correlation creates strong incentives for structuring because it implies homogeneous valuations of the loan pool as a whole, but heterogeneous valuations of its tranches. With disagreement about mean payoffs, in contrast, tranching is powerful whenever optimists cannot afford the entire loan pool, as it allows originators to concentrate optimist demand on the particular ranges of the payoff distribution where disagreement is strongest. Figure **??** shows that this is relevant for RMBSs, as investors tend to agree on the risklessness of their senior tranches.

With either kind of disagreement, the equilibrium return depends on the demand by investors with different beliefs. In equilibrium, the prices p_k of tranches $k = 1, ..., K$ have to be such that supply equals demand for every tranche, and that any investor $i = 1, ..., I$ expects to earn on all positive investments in her portfolio an equal return R_i not smaller than R, and greater than that she expects from any assets she does not hold. Solving the implied $(I + 1) \cdot K$ equilibrium conditions is complex. This subsection briefly illustrates the equilibrium return to tranching in a simple numerical example for the subprime RMBS considered in the left panels of Figures **??** and **??**, with five investor types that disagree about default correlation ρ_i^2 or the default probability $\overline{\pi}_i$. We normalize the mortgage supply and the mass of each investor type to 1 and set the endowment n equal to $\frac{3}{7}$ for all investors, such that at least three types are needed to buy the benchmark mortgage pool.

4.2.1 Disagreement about default correlation

We assume a uniform distribution, and consider two pairs of values $\{\rho^2_{min}, \rho^2_{max}\}$, corresponding to a 'weak' disagreement case (where $\{\rho^2_{min}$ and $\rho^2_{max}\}$ equal to 7 and 12 percent respectively), and a 'strong' disagreement case ($\rho^2_{min} = 2$ percent and $\rho^2_{max} = 16$ percent).[23] The return to tranching, defined as the difference in market values between the benchmark RMBS and that of the collateral when sold as a non-tranched, pass-through securitization, equals 44 (111) basis points in the weak (strong) disagreement case. This return is high because self-selection is strong: disagreement about valuations is concentrated in the equity, junior and mezzanine tranches, which are cheap and can thus be bought by a small number of specialized investors with 'extreme' beliefs of high correlation (for junior and equity tranches), or low correlation (for mezzanine tranches). The remaining investors are happy to buy the senior tranches at their 'consensus' valuation.

4.2.2 Disagreement about average default probabilities

To illustrate the determinants of the return to tranching with disagreement about mean payoffs, consider the same RMBS with a homogeneous perceived correlation equal to 9.5 percent (the mid-point of the weak disagreement case above), but a distribution of investors across $\overline{\pi}_i$ that is uniform in a range $[10.5, 14.5]$ percent around the benchmark value of 12.5 percent (with a 5-point, equally spaced, support and investor endowments as before). Because investors agree that the senior tranches are riskless the issuer can sell them to pessimists at the common valuation. Equity, junior and mezzanine tranches, in contrast, can be sold to optimists. A pass-through

[23] In the weak disagreement case, we choose a ρ^2_{min} equal to the maximum correlation compatible with an AAA rating of both senior tranches in our benchmark RMBS. This is to capture the intuition that structures were designed to give senior tranches top credit ratings, and that rating agencies often had optimistic assessments of default probabilities (?griffin2011). In line with this, ?ashcroft2010 (), p.13 find that the average fraction of subprime RMBS that received an AAA rating was 82 percent. We assume that the ρ^2_{max}-investors perceived a 0.5 percent probability of default rates in the RMBS pool reaching 40 percent or more, as observed in 2007 for US subprime mortgages (see, e.g., ?beltran2013 (), especially figure 4). In the second, 'strong' disagreement specification, we extend the range of values such that the ρ^2_{min}-investor would also just give the mezzanine II tranche an AAA rating, and such that the ρ^2_{max} investor perceives a probability of 1 percent of default rates rising to 40 percent or above.

securitisation, in contrast, requires three types to invest, and is thus priced by the median investor (with $\overline{\pi}_i = 12.5$ percent). The general equilibrium return to tranching thus equals the difference between the optimistic valuation and that of the median investor, in this case 113 basis points and thus approximately identical to that in the case of strong disagreement about default correlation.

5 Conclusion

Motivated by the strong, and in the case of GDP forecasts rising disagreement about the dispersion of outcomes in US surveys of investors and forecasters, this paper has looked at the role of collateralized asset trade in economies where investors disagree about risk, rather than mean payoffs as in the literature. A simple static model of investor disagreement showed how the introduction of simple collateralized debt allows investors who perceive high payoff dispersion to purchase upside risk by investing in a mortgage pool and using it to collateralize debt tranches that their low-risk counterparts value highly. A quantitative application to the market of US subprime RMBSs and CDOs showed how disagreement about the volatility of default rates, or the importance of aggregate factors for mortgage defaults, raises the price of junior RMBS tranches by between 40 and 110 basis points.

The theory presented in this paper has additional empirical predictions that can be compared to data even without information on the, typically unobserved, risk perceptions of investors.[24] For example, our mechanism requires that investors can issue non-recourse collateralized loans. It thus predicts an effect of heterogeneous risk perceptions on the price of private-label residential mortgage-backed securities (but not of seemingly government-guaranteed agency securitizations), or on house prices in jurisdictions with non-recourse residential mortgages (but not, or less so, in those with recourse mortgages such as some US states and most European countries).

[24] A case of observed disagreement is that about ratings, as documented by (?norden2014dynamics) in a large sample of US and European firms. It would be interesting to study empirically how credit rating disagreement affects asset prices.

Moreover, we would expect larger effects in markets where risk is important (in the sense of substantial default probabilities), and where disagreement about risk is likely to be stronger (such as for assets or contracts with a shorter history). Finally, we would expect the effect to increase over time both because disagreement about risk has seemingly increased (at least in the given sample of forecasters interviewed by the SPF), and because issuers of collateralized assets were increasingly able to draw on a more international and diverse investor pool. We leave formal empirical tests of these predictions to future research.

We also hope that our analysis opens some avenues for further theoretical research. We have abstracted from any additional dimensions of investor heterogeneity that may affect equilibrium asset prices. Importantly, it is sometimes argued that heterogeneity in risk appetite has encouraged the tranching of loan pools to create "safe" assets. In fact, ?allen1988optimal () show how a debt-equity financial structure can increase the financial value of firms (relative to equity-only financing) when investors have heterogeneous risk aversion. As Section B.1 in the Online Appendix shows in detail, general results are difficult to derive even with homogeneous risk aversion. This is because discounted payoffs do not typically inherit the stochastic dominance properties of undiscounted payoffs that allow to characterize the effects of disagreement theoretically.[25] In addition, a dynamic analysis, where risk arises both from future payoffs and price movements, seems particularly interesting,[26] as do concrete applications of the theory to other financial markets. Finally, an investigation into the sources of disagreement, or the determinants of risk perceptions, would be valuable.

Our results imply that investors on average make losses relative to their required rate of return. This suggests that there might be welfare-improving policy interventions that would

[25]Ellis et al. (2017) show how even with risk aversion the optimal equilibrium structure is such that tranches are sold to the investor with the maximum discounted expected payoff in equilibrium. They also show, however, that heterogeneous risk aversion as such may not encourage tranching (in an example with heterogeneou CARA preferences where heterogeneous portfolio shares of a risky asset imply homogenous valuations of any part of its cash-flow in equilibrium).

[26]The working paper version of this paper (?BroerKero2014) presents a simple example of a dynamic equilibrium in a scenario with learning that tries to capture the main features of the Great Moderation in the US. As a subset of investors adjusts their posterior estimate of volatility more quickly to the Great Moderation than the rest, increasing divergence of posteriors raises asset prices between 5 and 20 percent.

be interesting to study.[27] Moreover, the fact that disagreement about payoff dispersion makes investments more risky and raises leverage in the economy should be of interest for policy makers and regulators.

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6 Omitted proofs

Proof of Proposition ??.

(**??**) is simply the optimality condition (**??**) for choice of leverage $\bar{s}$. To understand (**??**), note that for any $p < E^s$ all agents would like to buy the mortgage pool, which cannot be an equilibrium. Equivalently, for any $p > \overline{p} \doteq E^s + E_L(min(s, \overline{s})) - E_H(min\{s, \overline{s}\})$ both type L and H investors would like to sell their endowment of the mortgage pool, again contradicting equilibrium. Type H optimality implies that they invest all resources in the pool using leverage when $E^s \leq p < \overline{p}$, but are indifferent between buying and consuming at $p = \overline{p}$. Thus, for $\overline{s}(\overline{p})$ the value of $\overline{s}$ that solves (**??**) when $p = \overline{p}$, if $\mathbb{B}(\overline{s}(\overline{p})) \geq \overline{p}$, type H's endowment is large enough to buy type L's endowment of the mortgage pool at the maximum price $\overline{p}$ that ensures her participation. There is thus an equilibrium price $\overline{p}$ at which H investors are happy to consume in period 0 any resources that remain after purchasing all of type L's endowment.

If for some price $p : E^s \leq p < \overline{p}$ it is true that $\mathbb{B}(\overline{s}(p)) < p$, H investors cannot buy the whole pool at that price but expect to make strictly positive profits $R_H^a > R$. They therefore invest all their resources, equal to $\mathbb{B}(\bar{s})$, to buy type L's endowment. Noting that E^s is a lower bound for the equilibrium price as argued above gives (**??**).

Finally, to prove uniqueness, since $\mathbb{B}(\bar{s})$ is trivially strictly upward-sloping, it suffices to show that (**??**) is downward-sloping. This follows by differentiating (**??**) totally

$$\frac{dp}{d\overline{s}} = -\frac{\frac{d\mathbb{C}}{d\overline{s}}}{\frac{d\mathbb{C}}{d\overline{p}}} \tag{14}$$

Weak concavity of $R_H^a(\overline{s})$ at the optimum choice of $\overline{s}$ implies that the numerator is weakly negative. Since $\frac{d\mathbb{C}}{d\overline{p}} < 0, \forall p, \overline{s}$ the result follows.

Proof of Proposition ??.

Note that under condition 2 we have for $\bar{s} = E^s$

$$\begin{aligned} \mathbb{B}(E^s) \quad &> \quad E^s - E_L(min\{s, E^s\}) - E_H(min\{s, E^s\}) + 2E_L[\min\{s, E^s\}] \\ &\geq \quad E^s + E_L[\min\{s, E^s\}] - E_H(min\{s, E^s\}) = \overline{p} \end{aligned} \tag{15}$$

which implies that even at the lower bound of leverage $\bar{s} = E^s$ H investors have resources larger than the value of assets evaluated at any $p \leq \overline{p}$. So equilibrium requires H investors to be indifferent between consuming and investing in leveraged assets, implying an equilibrium price $p = \overline{p} = E^s + E_L(min(s, \overline{s})) - E_H(min\{s, \overline{s}\})$. In turn, this implies $F_L(\overline{s}) = F_H(\overline{s})$ from (**??**), so $\overline{s} = E^s$ from single-crossing. Since E^s does not change in response to $d\theta_i$, neither does $\overline{s}$. But $\overline{p}$ rises with any mean preserving spread $\{d\theta_L > 0, d\theta_H < 0\}$ as $\frac{\delta E_L[min\{s,E^s\}]}{\delta\rho_L} \geq 0$ and $\frac{\delta E_H[min\{s,E^s\}]}{\delta\rho_H} \leq 0$.

To see how condition 1 implies the result, note that for any distribution that is not right-skewed, and therefore does not have more mass to the left of the mean than to the right (implying $1 - F_H(E^s) = 1 - F_L(E^s) \geq \frac{1}{2}$) we have $\underline{n_H} = E^s - E_L(min\{s, E^s\}) - E_H(min\{s, E^s\}) \leq$

$E^s - 2E_H(min\{s, E^s\}) \leq E^s - 2E^s(1 - F_H(E^s)) \leq E^s - 2E^s\frac{1}{2} = 0$. So condition 1 implies condition 2 holds for all n_H.

To see how condition 3 implies the result, note that when condition 1 is violated we have $p = p^\star < \overline{p}$ and $R_H^a > R$. (**??**) then implies $F_H(\overline{s}) < F_L(\overline{s})$, so $\overline{s} > E^s$. Thus belief divergence weakly raises $\mathbb{B}(\overline{s})$ at the original level of riskiness $\overline{s}$ as a mean preserving contraction dv_L increases the asset price through an increase in the price of loans $E_L[\min\{s, \overline{s}\}]$ for any $\overline{s} > E^s$. Since $E_L[\min\{s, \overline{s}\}]$ is increasing in $\overline{s}$, this implies the result whenever $\overline{s}$ does not fall in response to belief divergence.

To show that the optimal riskiness $\overline{s}$ rises with belief divergence, take the total differential of (**??**) (which exists by the definition of $d\theta_i$), and exploit the constant default probability $\frac{1-F_i}{d\theta_i} = 0, i = H, L$ to get

$$\frac{d\overline{s}}{dv_i} = \frac{-1}{\frac{d\Pi_H^{a2}}{d\overline{s}^2}} \left[\frac{\delta R_H^a}{\delta \rho_i}\right] > 0, i = H, L \tag{16}$$

where the inequality follows from $\frac{\delta R_H^a}{\delta \rho_i} > 0, i = H, L$ and $\frac{d\Pi_H^{a\,2}}{d\overline{s}^2} < 0$, the second order condition for $\overline{s}$ that maximizes type H profits Π_H^a. ■

Appendix for online publication only

A Additional Empirical Evidence

A.1 Disagreement in SPF forecasts - alternative measures

Our analysis of disagreement among SPF forecasters in Section **??** used a normal interpolation of forecasters' reported histograms. This allowed us to exactly decompose the measure of total disagreement into contributions of heterogeneous means and standard deviations. Figure **??** compares the disagreement measures in Figure **??** (dashed lines)) to an alternative without the normality assumption (solid lines). Specifically, the figure shows time series for the cross-sectional standard deviation of individual forecast dispersion (equal to the standard deviation of the forecast distribution), the cross-sectional standard deviation of forecast means, and the measure of total disagreement calculated as in (**??**), under both assumptions. All three measures are very similar to the benchmrk measures. Disagreement about the dispersion of GDP growth in the top panel is somewhat higher in levels than our benchmark measure, while total disagreement is somewhat lower. Importantly, however, the alternative measures correlate very strongly with those used for the main analysis.

A.2 Disagreement about US house price growth

This section complements the empirical analysis in the main by quantitfying disagreement about the dispersion of future house prices among US home owners. This is important because one of the most common kinds of collateral for debt products is real estate. US private homes in particular collateralized a large fraction of the structured securitizations that experienced a huge boom-bust cycle around the recent financial crisis. This section briefly presents some evidence on disagreement about future growth in (average) US house prices. Unfortunately, data on house price expectations is not available for the period prior to the crisis, and most more

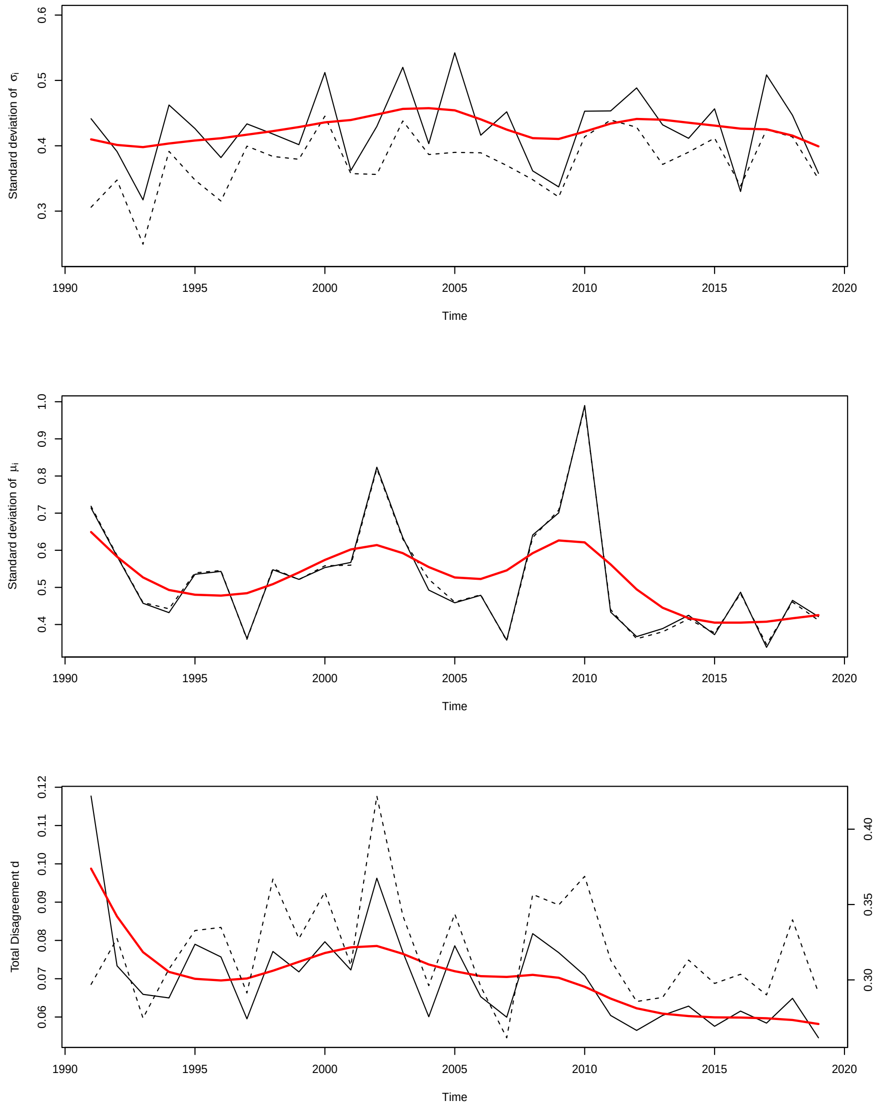


Figure 5: The top-left panel plots the time series of the standard deviations of forecast-standard deviations σ_{it} in the SPF, using a normal approximation to the forecast distribution as in Giordani and Söderlind (2003). The top-right panel plots the standard deviation of means μ_{it}. To remaining four panels show the total disagreement measure d (center-left panel), the contribution of heterogeneous forecast standard deviations σ_{it} (center-right panel), and of heterogeneous forecast means (bottom-left) panels, as well as the percentage of disagreement accounted for by those two parameters (bottom-right panel). The reds lines in the first 5 panels show the trend from an HP filter with smoothing parameter 25 (to adjust for the annual frequency, see Ravn and Uhlig (2002)). We omit two observations at the beginning and end of the sample to reflect the two-sided nature of the filter.

recent surveys only ask respondents for their average expected price growth.[28] One exception is the Federal Reserve Bank of New York's monthly Survey of Consumer Expectations, whose respondents are asked to indicate a histogram of their perceived distribution of the growth in average US home prices over the following 12 months.[29] This data can be used to document the heterogeneity in home owners' house price expectations. For example, the average expected house price growth showed a slightly decreasing trend around 4 percent during the sample period from June 2013 to December 2016.[30] At the same time, an average interquartile range of 3.9 percentage points indicates substantial heterogeneity in mean expectations. Importantly for this paper, the data also shows substantial heterogeneity in the perceived dispersion of house price growth: on average over the sample period 10 percent of respondents expect interquartile ranges larger than 6.4 percentage points, while another 10 percent expect them to be smaller than 1.1 percentage points. Using the same procedure as in Figure **??**, heterogeneity in means and standard deviations contributed by almost exactly equal amounts to overall disagreement about house price growth during the year ahead.

The SCE data are indicative of strong disagreement among consumers about both the mean and the dispersion of future house prices. The model we present in the next section implies that disagreement about house price dispersion may per se contribute to a higher level of house prices, as those who perceive upside risk value the option value of houses financed by non-recourse mortgages. Unfortunately, however, the available surveys contain no information about investor expectations about US house price growth before the financial crisis, and are thus of no help in calibrating our quantitative model of structured securitizations backed by US mortgage collateral

[28] The monthly Michigan Survey of Consumer Sentiment's series of expected home price changes starts in March 2007. The quarterly Zillow / Pulsenomics survey of economists, investment strategists, and housing market analysts has an even shorter history.

[29] Source: Survey of Consumer Expectations, ©2013-2017 Bank of New York (FRBNY). The SCE data are available without charge at http://www.newyorkfed.org/microeconomics/sce and may be used subject to license terms posted there. FRBNY disclaims any responsibility for this analysis and interpretation of Survey of Consumer Expectations data. The exact wording of the question we focus on is: "And in your view, what would you say is the percent chance that, over the next 12 months, the average home price nationwide will increase / decrease by x % or more."

[30] We exclude respondents that do not own their primary residence, who only reported a point forecast, and those whose expected growth rate differed from that implied by their reported histogram by more than 5 percentage points.

in Section ??.

B Extensions of the benchmark environment: risk aversion and options trade

This section discusses two self-contained extensions to the benchmark environment of Section ??.

B.1 Risk-averse investors

Like almost all studies of investor disagreement, including those where leverage amplifies payoff risk, such as Geanakoplos (2010), Fostel and Geanakoplos (2012), or Simsek (2013), our benchmark results are derived under the assumption that investors maximize expected profits, and are therefore risk-neutral. In this section we show how the main result, that leveraged asset trade increases asset prices when investors disagree about risk, also holds in a version of the environment with risk aversion.[31] As we will see, however, with risk-averse preferences, the quantitative effect of disagreement about payoff risk on the price of collateral assets is decreasing in the asset supply as agents are more reluctant to leverage aggregate risks that comove strongly with consumption, as opposed to idiosyncratic risks.

Consider the two-type environment of Section ?? but with preferences that have constant relative risk aversion equal to γ[32]

$$U = u(c) + \frac{1}{R}u(c'),\ u(c) = \frac{c^{(1-\gamma)} - 1}{1-\gamma} \tag{17}$$

We continue to abstract from discounting for simplicity by setting $R = 1$, and normalize the

[31]The analysis is related to Dow and Han (2016), who prove existence of equilibrium in a similar environment with heterogeneity in endowments and give numerical examples for asset price overvaluation.

[32]We make this assumption partly for simplicity. A well-behaved utility function with a strictly positive third derivative is sufficient for the analytical results below.

endowment of consumption goods in the first period to $n_i = 1, i = L, H$. Moreover, for tractability, we assume that type $i = L, H$ expects payoffs to follow a uniform distribution on a support $[1 - \epsilon_i, 1 + \epsilon_i]$, with $\epsilon_H > \epsilon_L > 0$.

Both agents solve a version of problem (**??**) adjusted for the risk-averse preferences (**??**), and the additional investment opportunities (non-collateralizable assets and storage). A general competitive equilibrium is then a set of prices $\{p^1, p^2, q(\bar{s})\}$, consumption plans c_j and c'_j, storage d_j and financial portfolios $\{a_j^1, a_j^1, b_j(\bar{s})\}$ for both types $j = L, H$, such that all agents solve their problem at given prices subject to their budget set, non-negativity constraints on storage, short-sale constraints on assets, as well as collateral constraints on loans, and the markets for securities and consumption goods clear.[33] Note that the price and quantity of collateralized loans are, again, a function of their face value $\bar{s}$.

The introduction of risk aversion changes the analysis in two important ways: First, risk-neutral agents concentrate all their investments in the highest yielding opportunity, whose equilibrium price equals investor resources per unit of supply, or its expected value discounted at the rate of time preference, whatever higher. With risk aversion, in contrast, and in the absence of second period income, the ability to invest in alternative, particularly (lower-yield) riskless assets, is important for collateralization incentives.[34] We thus assume that agents can transfer resources to period 1 through one of three ways: as before, they can issue an amount b_j in loans by providing collateral (as there is no commitment to promises) and buy the exogenous asset. In addition, however, we add the possibility to invest d_j units in a storage technology with gross return of R, normalized to 1.

A second difference arises because, unlike with risk neutrality, where expected payoffs discounted at the rate of time preference E^s are an obvious benchmark value of the asset, with risk aversion there is no such 'fundamental value'. To characterize how disagreement about payoff risk affects asset prices with and without trade in risky collateralized loans, one can either study

[33] See (Geanakoplos and Zame, 2014) for an equivalent definition of competitive equilbrium in a more general economy with collateral constraints, as well as a proof of existence of equilibrium.

[34] Trivially, with CRRA utility, agents will never leverage their whole asset holdings using risky loans when they have no other claims to future consumption.

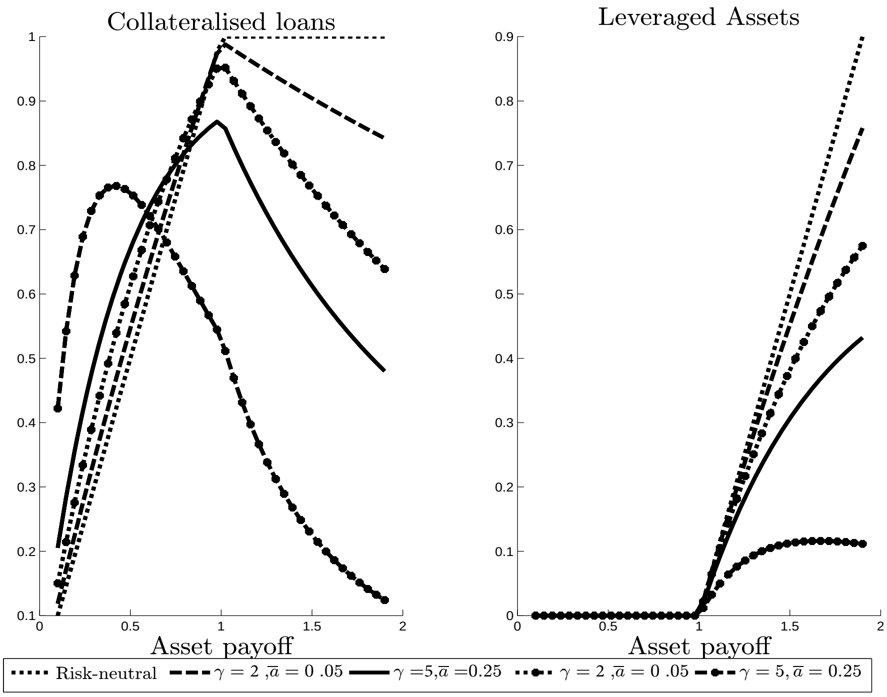
the relative price of collateralizable and non-collateralizable assets in the equilibrium of an economy where both are traded (which we call the 'within-economy collateral premium', studied, for example, in Dow and Han (2016)), or across equilibria of economies with different collateralization possibilities (the 'cross-economy collateral premium', as in Allen and Gale (2000)'s analysis of price bubbles due to limited liability). Below, we characterize the within-economy premium analytically, and provide quantitative examples for the cross-economy premium. For this we assume that agents are endowed with two kinds of assets 1 and 2 whose quantities satisfy $\overline{a}^1 + \overline{a}^2 = \overline{a}$. Both assets have identical payoffs but only asset 1 can be used as collateral for loans (for example because payoffs from asset 2 are observable only to the owner). We denote as a_j^i type j's holdings of asset i at the end of the first period, as $a_j = a_j^1 + a_j^2$ j's total asset holdings, and as p^i the equilibrium price of asset i.

Figure **??** illustrates how risk aversion changes the incentives to trade collateralized loans. As a function of (uniformly distributed) asset payoff $s \in \{0.1, 1.9\}$, it depicts payoffs of collateralized loans (in the left panel) and leveraged assets (in the right panel) discounted using type H's 'autarky pricing kernel', equal to her stochastic discount factor when optimally choosing storage but keeping her asset endowment $\overline{a}$ unchanged, for $\overline{s} = 1$ and different values of $\overline{a}$ and risk aversion γ.[35] For low asset endowment and low risk aversion, the discounted payoffs approximately equal those under risk-neutrality. Collateralized loan payoffs are thus a concave function of the underlying asset payoff s while leveraged asset payoffs follow a convex function. At high values of risk aversion, and with large asset holdings $\overline{a}$, however, the shape of the pricing kernel, which follows a declining and (with CRRA preferences) convex relation with consumption in the second period, affects more strongly the relationship of discounted payoffs with the underlying payoff s. Apart from increasing precautionary savings (and thus reducing the level of the discounted payoffs, which otherwise would all cross the point $(1, 1)$ in the left panel), the fact that low payoffs become more valuable relative to high payoffs has two effects: First, it reduces the average discounted payoffs of leveraged assets (which pay nothing below $\overline{s}$) relative to those

[35]Given their identical endowments these 'autarky valuations' are similar for both types (and only differ because of small differences in precautionary storage), but their expectations differ because of disagreement about payoff dispersion.

of collateralized loans. Second, discounted payoffs no longer inherit the convexity and concavity properties from the risk-neutral case. The first effect dampens the impact of disagreement on asset prices through leveraged asset trade when risk aversion is high and asset supply large. The second effect makes it, essentially, impossible to derive general analytical results for the environment with risk aversion. With uniform payoffs, we can, however, show that H investors always buy assets using leverage, and that the within-economy collateral premium is strictly positive for exogenous and extreme levels of leverage.

Figure 6: Discounted asset payoffs in autarky with risk aversion



As a function of asset payoffs $s \in [0.1, 1.9]$ along the bottom axis, the figure plots the discounted 'autarky' payoffs of collateralized loans $\frac{u'(c'(s))}{u'(c)} \min\{s, \overline{s}\}$ and leveraged assets $\frac{u'(c'(s))}{u'(c)} \max\{s-\overline{s}, 0\}$ when agents choose their storage optimally but keep their asset endowment $\overline{a}$ unchanged, for $\overline{s} = 1$ and different values of relative risk aversion γ, and two levels of the asset endowment $\overline{a}$ whose expected payoffs equal, respectively, 10 and 40 percent of average per-period consumption.

B.1.1 Equilibrium leverage and the within-economy collateral premium

To derive analytical results, we assume that assets are endowed to outside agents who derive utility only from first period consumption, as in Simsek (2013). This eliminates wealth effects of price changes. We return to the standard assumption of asset endowments in the quantitative analysis.

Proposition 4 - *Leveraged asset trade in equilibrium*

Consider an economy where asset 1 *is traded and collateralized loans of face values* $1-\epsilon_L$ *and* $1+\epsilon_L$ *are available. In any equilibrium, type* H *always buys a strictly positive amount of asset* 1 *and uses at least part of it as collateral for loans.*

Proof of proposition ??.

In the absence of leverage, both agents are indifferent between assets 1 and 2. Type H always purchases a strictly positive total amount of assets $a_i > 0$ that is strictly smaller than that purchased by type L.[36] When collateralized loans are available, there are deviations from this portfolio that type H perceives as strictly profitable. First, if type L stores a positive amount, type H can issue at least a small positive quantity of collateralized loans with unit face value $\overline{s} = 1 - \epsilon_L$. Type L perceives this loan as riskless and is happy to substitute it at the unit price $1 - \epsilon_L$ for (part of) her storage. Storing the proceeds in a separate account, type H's payoffs strictly dominate those of the original portfolio, as she earns an additional amount $(1-\epsilon_L) - s^i_H > 0$ whenever payoffs are below $1 - \epsilon_L$.

If type L does not store, type H can still offer to buy some of her asset holdings in exchange of a collateralized loan with face value $1 + \epsilon_L$. Type L is exactly indifferent between the asset and that loan, which she perceives to have identical payoffs since, for her, $min\{s, 1+\epsilon_L\} = s$. Type H, however, expects to receive a net payment of $s - (1+\epsilon_L)$ for high realizations of $s > 1 - \epsilon_L$. There is thus no equilibrium without leverage. ■

The proof of proposition **??** shows that there is no equilibrium without collateralized loans as, in any such equilibrium, type H would perceive a profitable deviation from selling collateralized loans to type L that the latter either perceives as riskless ($\overline{s} = 1-\epsilon_L$) or payoff-equivalent to the

[36] To see this, assume that either type holds 0 assets. Her portfolio is thus riskless, and she values the asset at the fundamental value E^s, which exceeds any price the second type will be happy to pay when holding the whole asset supply, independent of her amount of storage. A similar contradiction can be derived assuming $a_L <= a_H$.

asset itself ($\overline{s} = 1 + \epsilon_L$). The following proposition shows, however, that even with equilibrium leverage there is no within-economy collateral premium unless type H buys the whole supply of collateral assets using leverage.

Proposition 5 - *Within-economy collateral premium I*

Consider the economy where both assets are traded ($\overline{a}^1 > 0, \overline{a}^2 > 0$). There is no within-economy collateral premium unless H investors buy the whole supply of asset 1 using leverage.

Proof. Note that type L agents would never use leverage to buy asset 1. Thus, they buy asset 1 only at a price that does not exceed that of asset 2. Moreover, even when type H holds the whole supply of asset 1 but does not use all of it as collateral, arbitrage between assets 1 and 2 using type H's equilibrium pricing kernel implies $p^1 \leq p^2$. ■

Since it is difficult to characterize equilibria where $\overline{s}$ is optimally chosen as in the benchmark case, the remainder of this section takes as given a single exogenously given level of leverage $\overline{s}$.

Proposition 6 - *Within-economy collateral premium II*

Consider the economy where both assets are traded. The equilibrium price of asset 1 strictly exceeds that of asset 2 if either of the following conditions holds

1. *$\overline{s} = 1 + \epsilon_L$ and the endowment of asset 1 does not exceed that of asset 2 ($\overline{a}^1 \leq \overline{a}^2$).*

2. *$\overline{s} = 1 - \epsilon_L$ and asset endowments are small in the sense that $b_L > (1 - \epsilon_L)\overline{a}^1$ and $a_H > \overline{a}^1$.*

Note that, since storage b_L approaches $\frac{1}{2}$ as $\overline{a}$ approaches 0, and since $a_L > 0$ and $a_H > 0$ whenever $\overline{a} > 0$, there are numbers $\overline{a} > 0$ and $\overline{a}^1 > 0$ that fulfill the condition in 2.

Proof of proposition ??.

Ad 1. We show how, in any equilibrium with leverage at $\overline{s} = 1 + \epsilon_L$, type L agents hold collateralized loans and non-collateralized assets that they regard as payoff equivalent, implying $q = p^2$. Type H's perceived net profits from leveraged asset purchase then imply $p^1 > p^2$.

In the absence of leverage, both assets trade at the same price p^{nc} and type L agents, who hold a larger quantity of assets than type H, can buy the whole supply of asset 1 since $a^0 > \frac{1}{2}\overline{a} \geq \overline{a}^1$. Suppose instead type H buys the entire supply of asset 1 financed by collateralized loans of face value $1+\epsilon_L$. Type L agents see those loans as payoff-equivalent to both assets. Whenever $min\{q, p^1, p^2\}$ does not exceed p^{nc}, their combined demand of assets and loans thus strictly exceeds the total quantity of collateralized loans (equal to $\overline{a}^1$). Moreover, since demand for non-collateralized assets by H investors is strictly reduced by their leveraged asset purchases (whose net payoff is perfectly correlated with non-collateralized asset payoffs for $s > 1-\epsilon_L$), we can rule out a price of asset 2 above p^{nc}. Market clearing together with type L's arbitrage condition for assets and loans thus requires $p^2 \geq q$ and $p^1 \geq q$ with at least one equality. H investors' optimality condition and market clearing for asset 1 imply

$$p^1 = E_H[\frac{U'(c'_H)max\{s-(1+\epsilon_L)\}}{U'(c_H)}] + q > q \tag{18}$$

implying $p^2 = q$ and $p^1 > p^2$.

Ad 2. When $a_H > \overline{a}^1$ H investors can buy the entire stock of asset 1 and collateralize it using loans issued at $\overline{s} = 1-\epsilon_L$ to type L agents. The price of this loan is simply $q = 1-\epsilon_L$ from arbitrage with type L's remaining storage investment, which is positive since $b_L > (1-\epsilon_L)\overline{a}^1$. H investors' optimality condition and market clearing thus imply

$$\begin{aligned} p^1 &= E_H[\frac{U'(c'_H)max\{s-(1+\epsilon_L)\}}{U'(c_H)}] + 1 - \epsilon & (19)\\ &= E_H[\frac{U'(c'_H)}{U'(c_H)}s] + E_H[\frac{U'(c'_H)}{U'(c_H)}max\{(1+\epsilon_L)-s, 0\}]] > p^2 & (20) \end{aligned}$$

where the last inequality follows because the payoff of a marginal unit of asset 1 perceived by type H dominates that of asset 2 for every state $i = 1, ..., N$ (and strictly so for states where $s < 1-\epsilon_L$). Since type H's pricing kernel is strictly positive, this implies a higher equilibrium price of asset 1. ■

The proof exploits the fact that, for a sufficiently small supply of collateralizable asset 1 and either of the two extreme levels of $\overline{s}$, type L is happy to substitute her investments in either storage or outright asset purchase with the entire collateralized loan supply at an unchanged expected return. At the price that prevailed in the absence of leverage, H's expected return from the marginal unit of leveraged assets thus exceeds its costs, implying an increase in the equilibrium asset price.

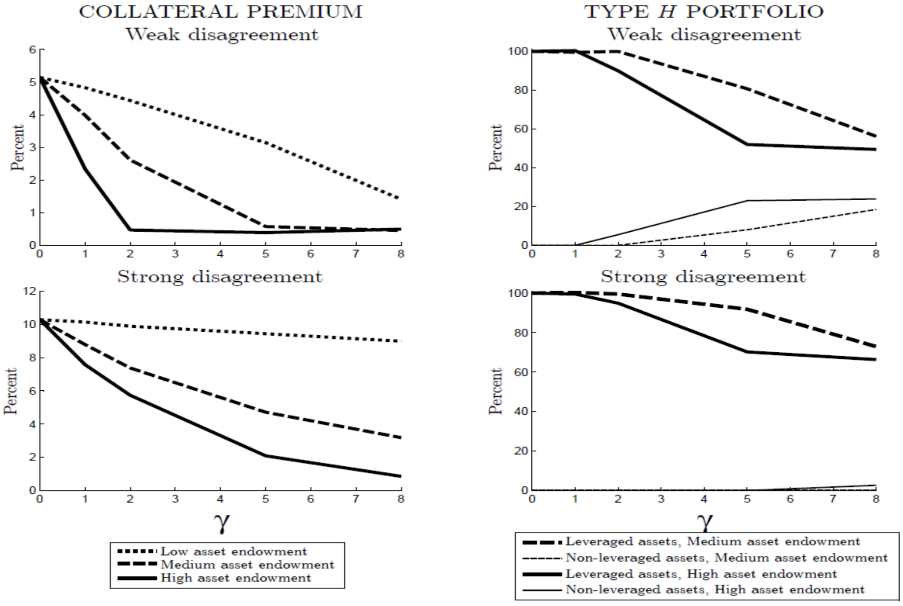
Note that all results in this section hold when we relax the assumption of uniform distributions, as long as the support of payoffs perceived by type H has upper and lower bounds that are, respectively, strictly greater and lower than those of type L agents.

B.1.2 A quantitative analysis of the cross-economy collateral premium

The analytical results focused on extreme values of leverage equal to the bounds of type L's perceived payoff distribution and the resulting within-economy collateral premium. This is because it is difficult to theoretically study intermediate leverage levels, or to characterize the cross-economy collateral premium, equal to the relative asset price in identical economies with and without collateralized loan trade. Since it is similarly difficult to compute within-economy collateral premia, requiring the solution to a four-asset general equilibrium portfolio problem, with accuracy, the rest of the section looks at the cross-economy collateral premium in several quantitative examples of risky collateralized lending when leverage equals the intermediate value $\overline{s} = 1$, the optimal face value with risk-neutrality under assumption that type H is cash-rich (see condition 2 in Proposition **??**). For this, we set $\epsilon_H = 0.9$ and look at two different values for ϵ_L such that the standard deviation of asset payoffs s perceived by type L is, respectively, 20 percent ('weak disagreement') and 40 percent ('strong disagreement') lower than that of type H. To highlight the importance of the portfolio share of risky assets, we set $\overline{a}$ such that expected asset payoffs approximately equal 2.5, 20 and 40 percent of average per-period consumption, which we call 'low', 'medium' and 'high' asset supply, respectively.[37] For different values of risk aversion γ (along the bottom axis), Figure **??** depicts in its left hand column the cross-economy collateral

[37]We focus on the share of average consumption as it is unaffected by equilibrium prices. The corresponding values of $\overline{a}$ are 0.0125, 0.1, and 0.25 respectively. To compute the equilibria, we use discrete uniform distributions with 7 support points.

Figure 7: Cross-economy collateral premium and portfolios with risk aversion



For different values of risk aversion γ along the bottom axis the figure presents, in its left hand column, the cross-economy collateral premium (calculated as the percentage difference of asset prices in an economy where the whole asset stock can be used as collateral for loans and those in an economy without collateralization) and, in its right hand column, the leveraged and non-leveraged assets held by type H as a percentage of the total asset supply.

premium, calculated as the percentage difference of asset prices in an economy where the whole asset stock can be leveraged ($\overline{a}^1 = \overline{a}$) and those in an economy without collateralization ($\overline{a}^2 = \overline{a}$). With risk-neutral agents, the premium is independent of the economy's asset supply (as all cases we look at fulfill assumptions **??** and condition 2 in Proposition **??**)), and at around 10 percent about twice as large under strong disagreement (in the bottom left panel) compared to weak disagreement (in the top left panel). As suggested by Figure **??**, rising risk aversion γ reduces the attractiveness of leveraged assets, and thus the collateral premium. And, as expected, for a large asset supply the premium declines faster (to about a tenth of its risk-neutral value at $\gamma = 8$) than at low endowments (where the premium at $\gamma = 8$ is still 90 percent of its risk-neutral value in the case of strong disagreement). In fact, H investors are increasingly less willing to hold a large stock of assets using leverage at higher risk aversion. As the right-hand panels of Figure **??** show, when the incentives to diversify their portfolio rise with γ, H investors eventually invest in assets without leverage. Their leveraged investments decline faster than non-leveraged investments rise, however. This is because type L agents start buying the asset as its price drops with rising risk aversion. Importantly, even when both agents buy a positive quantity of assets without leverage, the cross-economy collateral premium remains positive. This is in contrast to the within-economy collateral premium, which, according to proposition **??**, drops to zero in this case. In fact, at high values of γ, a small quantitiy of asset 2 would achieve the same price as asset 1. This price, however, exceeds that in an economy without collateralization: using part of her asset holdings as collateral for loans sold to type L agents makes type H's consumption less sensitive to downside payoff risk and thus increases her valuation even of non-leveraged assets.

B.2 Options trade

In the benchmark environment, the equilibrium price of assets is elevated because of their collateral value: collateralized loans are the only means of exploiting perceived gains from trading upside and downside risk. Collateralization can thus be viewed as a substitute for trade in simple (European) options, whenever these are not available, too costly or simply not used. This section shows, however, that with disagreement about risk, collateralization continues to be used for speculative purposes and collateral prices continue to include a premium even when options are available and traded. The size of the premium, however, depends crucially on collateral requirements for options.[38]

When there is no other collateral than the exogenous asset, put options, whose payouts are high when assets pay little, cannot be collateralized. Call options, in contrast, pay in high payoff states and can thus be collateralized by the asset or any other claim collateralized by it. In equilibrium, type L agents thus optimally use their loan portfolio as collateral for issuing call options, whose payouts they expect to be low. As is easy to show formally[39], this raises

[38] To see how trade in collateralized loans and options imply the same payoffs, note that issuing a put option with strike price s^p collateralized by s^p units of riskless debt is equivalent to buying risky debt with face value s^p collateralized by the asset: both have payoffs equal to $min\{s, s^p\}$ in period 1. Similarly, buying the asset using a collateralized loan of face value $\overline{s}$ as leverage has the same stochastic payoff as buying a call option with strike price $s^c = \overline{s}$, or - due to to 'put-call-parity' - as buying the asset plus a put option with equal strike price and issuing riskless debt equal to $\overline{s}$.

[39] Results for this case are contained in a previous draft that is available from the authors upon request.

the expected return on collateralized loans, the equilibrium loan price q and ultimately the price of the collateral asset above its value in the absence of options trade. That options trade increases asset prices is, in fact, not surprising: when more complex contracts allow agents to better exploit perceived gains from trade, collateral for financial trade becomes more valuable, implying a higher equilibrium price of collateral assets.

The rest of this section concentrates on trade in options when there is an additional cash technology to collateralize them. Typically, this widening of the collateral pool reduces the price of other collateral assets through collateral arbitrage. In our environment, however, cash is an inefficient form of collateral for call options relative to the asset itself: while, for any given strike price, one unit of the asset always suffices to collateralize a call option, the cash collateral requirement rises one-for-one with the maximum asset payoff. Proposition **??** shows how this implies that, when cash is not too unequally distributed among investor types, the asset price continues to include a premium, similar to the equilibrium without options trade.

This section sets $s^{min} = 0$ and $R = 1$ to ease notation. In addition to the exogenous asset and collateralized loan of Section **??**, agents can also trade simple (European) put and call options, whose issuer receives a fee p_c and p_p in return for payments equal to $max\{s - s_c, 0\}$ and $max\{s_p - s, 0\}$, respectively, in period 1, where s_c and s_p are the strike prices of the options. The equilibrium definition is the same as that in section **??**, amended to include two additional assets with associated collateral requirements. Suppose there is a cash asset, or storage technology, which returns $R = 1$ units of consumption in period 1 for 1 unit of consumption invested in period 0 and can be used as collateral for options trade. Importantly, this eliminates the kind of cash-rich equilibria we focused on in the previous section, because the consumption endowment now represents not just asset demand, but also collateral supply. Option prices are thus a function of supply (through type L's endowment) and demand (type H's endowment). Depending on their relative size, the perceived gains from trading them accrue to the issuer, the buyer, or both, implying that the average expected portfolio return $R_i, i = L, H$ of one or both

There, we also provide an example where trade in call options doubles the equilibrium collateral premium.

agents exceeds their discount factor $R = 1$. The exogenous asset supply continues to be used as collateral either for call options or loans, which yields higher returns than buying assets outright. Prices are determined by 'collateral arbitrage', such that agents are indifferent between posting cash or the asset as collateral.

We show how the asset price continues to include a premium, as the returns on the exogenous asset $\frac{E^s}{p}$ are lower than those on options as long as both agents share the perceived gains from trade (equivalent to $R_L, R_H > R$). Asset prices may or may not, however, exceed the fundamental value E^s. Intuitively, the cash required to collateralize options is a function of the maximum payoffs only, equal to either s^p (for put options, since we normalize the lower bound $V_{rec} = 0$) or $s^{max} - s^c$ (for call options). The value of the collateral asset, however, depends on its whole payoff distribution, not just the extremes. The higher the probability of low asset payoffs (which reduces the fundamental asset value) and the wider the range of payoffs (which increases the required amount of cash collateral), the cheaper the exogenous asset becomes as a form of collateral relative to cash. The proof of a positive asset price premium even with options trade is complicated by the fact that strike prices are endogenous and may differ between the four assets (cash-collateralized calls and puts, asset-collateralized loans and calls). We therefore use a 'revealed portfolio preference' argument: whenever cash-collateralized call and put options are traded, their return must not be lower than that on asset-collateralized calls or loans respectively, which - at the same strike prices - would have the same period 1 payoffs. This bounds the price of the asset from below.

Proposition 7 - *Asset prices with option trade and cash collateral*

When cash-collateralized put and call options are traded, and endowments are such that perceived gains from trade are shared (in the sense that both agents expect positive returns $R_L, R_H > R = 1$), the asset price exceeds discounted values expected by either type: $p > \frac{E^s}{R_i}, i = L, H$.

Proof of proposition ??.

Suppose cash-collateralized put options are traded. A put option with strike price s^p collateralized by s^p units of cash gives a type L issuer a period 1 payoff equal to $min\{s, s^p\}$ and requires a net injection of cash collateral equal to $s^p - p^p > 0$. Type H can thus always sell type L at price $q = s^p - p^p$ a collateralized loan of face value s^p that yields the same period 1 payoff $min\{s, s^p\}$. The resulting expected return must not be larger than the portfolio return expected from type H's actual investments (which include the put option by assumption): $R_H \geq \frac{E_H[max\{s,s^p\}]}{p-q}$. This puts a lower bound on the asset price

$$\begin{aligned} p &\geq \frac{E_H[max\{s - s^p, 0\}]}{R_H} + s^p - p^p \\ &= \frac{E_H[max\{s - s^p, 0\}]}{R_H} + s^p - \frac{E_H[max\{s^p - s, 0\}]}{R_H} \\ &= \frac{E^s}{R_H} + \frac{R_H - 1}{R_H} s^p \end{aligned} \tag{21}$$

where the second line follows since H investors expect a return from put options equal to their portfolio return $R_H = \frac{E_H[max\{s^p - s,0\}]}{p^p}$ and the third from 'put call parity' at the common strike price s^p. The asset price thus strictly exceeds type H's 'fundamental valuation' $\frac{E^s}{R_H}$ whenever $R_H > 1$.

By a similar reasoning, when cash-collateralized call options are traded at a strike price s^c, the return from issuing them using $1 - s^c$ units of cash as collateral must equal type L's expected portfolio return: $R_L = \frac{E_L[max\{1-s,1-s^c\}]}{1-s^c-p^c}$. Asset prices must be such that the type L issuers do not strictly prefer to issue asset-collateralized calls whose return equals $\frac{E_L[min\{s,s^c\}]}{p-p^c}$. This again bounds the asset price

$$\begin{aligned} p &\geq \frac{E_L[min\{s, s^c\}]}{R_L} + p_c \\ &= \frac{E_L[min\{s, s^c\}]}{R_L} + \frac{E_L[max\{1 - s, 1 - s^c\}]}{R_L} + 1 - s^c \\ &= \frac{E^s}{R_L} + \frac{R_L - 1}{R_L}(1 - s^c) \end{aligned} \tag{22}$$

where the third line follows since $E_L[min\{s, s^c\} + E_L[max\{1 - s, 1 - s^c\} = E^s$. Again, $p > \frac{E^s}{R_L}$ whenever $R_L > 1$. ■

The following proposition shows, by example, how asset prices may be above their fundamental value E^s even when cash-collateralized put and call options that reference its payoff distribution are traded.

Proposition 8 - *Asset prices may exceed their fundamental value*

When cash-collateralized put and call options are traded, the asset price may exceed its fundamental level E^s.

Proof of proposition ??. The proof is by example. Suppose $S = \{0, 1, 2\}$ with $f_L = \{\frac{5}{8}, \frac{2}{8}, \frac{1}{8}\}$, $f_H = \{\frac{3}{4}, 0, \frac{1}{4}\}$, $n_L = \frac{11}{30}$ and $n_H = \frac{19}{30}$. Take expected portfolio returns $R_L = \frac{5}{4}$ and $R_H = \frac{15}{14}$, and an asset price $p = \frac{16}{30} > E^s = \frac{1}{2}$. To see how this is indeed an equilibrium, note that type L and H perceive $s^p = s^c = 1$ and $\overline{s} = 1$ as, respectively, optimal strike prices for options and an optimal face value of collateralized loans, because return functions all have a kink at $s = 1$.[40] Puts are then priced by type L agents such that

$$\frac{E_L[min\{1, s\}]}{1 - p^p} = \frac{5}{4} \tag{23}$$

implying $p^p = \frac{7}{10}$, and similarly $p^c = q = \frac{3}{10}$. Type H's expected return from buying cash-collateralized put and call options at these prices are $R^p_H = \frac{15}{14}$ and $R^p_H = \frac{10}{12}$ respectively, implying that cash-collateralized call options are not actually traded. Leveraged asset investments by H investors have an expected payoff equal to $\frac{E_H[max\{s-1,0\}]}{p-q}$ which implies an arbitrage asset price equal to $\frac{16}{30}$.[41] At the assumed endowments (plus the return from selling put options and their asset endowment equal to 1), H investors can exactly afford to issue 1 unit of put options and buy all 2 units of collateralized loans. Similarly, H investors can afford to buy all put options and all assets using leverage. ■

[40] Concentrating on put options and loans, since all assets issued are priced at the assumed portfolio returns, we have price functions $p^p = \frac{14}{15} E_H[max\{s^p - s, 0\}]$ and $q = \frac{4}{5} E_L[min\{s, \overline{s}\}]$. This implies an issuer return of $\frac{E_L[min\{s^p,s\}]}{s^p - \frac{14}{15} E_L[min\{s^p,s\}]} = \frac{\frac{1}{4} min\{s^p,1\} + \frac{1}{8} s^p}{\frac{6}{20} s^p}$ for put options and $\frac{E_H[max\{\overline{s}-s\}]}{p - \frac{4}{5} E_L[min\{\overline{s},s\}]} = \frac{\frac{1}{4}(2-\overline{s})}{p - \frac{4}{5}(\frac{1}{4} min\{\overline{s},1\} + \frac{1}{8}\overline{s})}$ for loans. The former is flat below and declining above $s^p = 1$. The latter is increasing below and decreasing above $\overline{s} = 1$.

[41] At this asset price and type H's valuation of a call option, type L expect a return from issuing asset-collateralized call options equal to their portfolio return. They are thus indifferent between issuing them or not. We look at an equilibrium where call options are not traded, other than implicitly through leveraged asset trade.

Note that the asset returns in the example crucially depend on relative endowments: it is easy to see that when $n_L \geq \frac{3}{5}n_H + \frac{2}{5}$ H investors' endowment is sufficiently small to lower option and loan prices to type L's reservation level, implying $R_L = 1$, and $R_H = 2$. In other words H investors harvest all perceived gains from trade. The reverse is true when type L's endowments are small enough, or $n_H \geq 3n_L$, implying $R_H = 1$, and $R_L = 2$. In both cases, the asset price equals its fundamental value: $p = E^s = E^s$.